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Three-dimensional object recognition using joint fractional Fourier transform correlators with the help of digital Fresnel holography

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ABSTRACT

Digital holography (DH) helps record true three-dimensional (3D) perspective of a 3D object. The technique of DH can be used for various applications, including 3D object recognition, which can be implemented optically. In this paper, 3D object recognition with joint fractional Fourier transform (JFRT) correlators is proposed in which the technique of digital Fresnel holography is used. Digital Fresnel hologram of a 3D object is synthesized and reconstructed digitally. After removal of the dc and twin image terms from the reconstructed image, two different target objects have been compared using linear and nonlinear JFRT correlators. The results are also verified using an optically recorded digital Fresnel hologram. The performance measure parameters, such as, discrimination ratio, peak-to-correlation energy, and peak-to-sidelobe ratio have been computed to compare the two proposed correlation schemes.

1. Introduction

Digital Fresnel holography is a holographic technique based on Fresnel diffraction of light wave [1]. Digital Fresnel hologram (DFH) is obtained as the interference pattern between a reference wave and Fresnel transform of an object wave. The DFH must be illuminated by the same reference wave and the reconstructed image of the original three-dimensional (3D) object is obtained at a distance at which the hologram was recorded. Reconstruction of the optical wavefield can be performed either numerically or optically [1,2]. Digital holography (DH) offers a lot of flexibility for improving the quality of the reconstructed wavefront. Employing the image processing algorithms, zero-order and twin images can be removed by applying various techniques [3–5]. There are various methods of numerically reconstructing the wavefront [1,6].

DH has found applications in the field of microscopy [7,8], 3D displays [9,10], metrology [11], etc. The 3D object identification is another area where DH is efficiently used [12–17]. Seifi et al. [12] proposed fast and accurate 3D object recognition with the help of directly matching the diffraction patterns which is recorded in low dimensional space to reduce the computational cost. Nelleri et al. [13] proposed a method for 3D object recognition in which Mexican-hat wavelet matched filter is used to enhance the discrimination ability. Phase-shifting digital holography is used to record the complex distribution of the 3D object in a single plane in Fresnel region and a digital hologram for 3D reference pattern is also recorded which acts as a correlation filter [14–17]. Two-dimensional (2D) correlators cannot be used to identify targets in 3D space because 2D correlators cannot determine longitudinal distances accurately. A 3D object should be processed along all the three dimensions [18]. However, different perspectives of a 3D object may be used to identify a 3D object using 2D correlators [19,20].

There are various methods of detection of 3D objects using digital holograms. Identification of 3D objects based on DFH can be performed using joint fractional Fourier transform correlator (JFRTC). In 3D correlator proposed by Rosen [19,20], 3D target and reference objects are observed simultaneously, and their different perspectives are displayed onto a spatial light modulator (SLM). The projected images are electro-optically processed to get the 3D correlations. A shifted correlation peak is observed for longitudinally displaced 3D objects. In this correlator, directedness and high processing speed of optical correlators are retained, unlike previously proposed 3D correlators [21]. The joint transform correlator (JTC) based on fractional Fourier transform (FRT) [22–25] has a property of shift-variance unlike conventional JTC. This property may be used in applications where a specific object must appear at a certain location. Performance of the correlator may be enhanced by introducing nonlinearity in joint power spectrum [26–28].

An optical correlator for identification of 3D objects is required for several applications including, recognition of harmful microbes and quality check for potable water. The 3D correlators should have the potential to work in real-time with high discrimination ratio.

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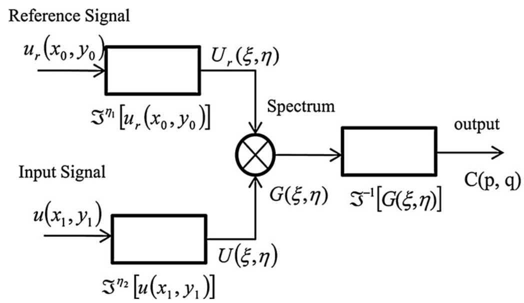

Fig. 1. Schematic of joint fractional Fourier transform correlator.

3D identification of micro-organisms can help prevent and control outbreaks of various diseases [29,30]. Correlation techniques based on DH can automatically focus at 3D volume under examination. Thus, combining the benefits of an optical correlator with DH can be applied for recognition of 3D microbiological objects. To develop such a correlator, in this paper, we have proposed an FRT based scheme of object recognition. DH of one perspective of two different 3D objects are simulated using digital Fresnel holography and corresponding 3D images of the objects are reconstructed numerically using Fresnel–Kirchhoff integral for free space propagation. The reconstructed images are then compared with the target 3D objects with the help of JFRTC and nonlinear JFRTC (NJFRTC). These schemes are also verified by experimentally recording DFH and compare the numerically reconstructed image with target images. Various performance measure parameters, such as, discrimination ratio, peak-to-correlation energy, and peak-to-sidelobe ratio are also computed to check the strength of the proposed optical correlation schemes [31,32].

2. JFRTC

Fig. 1 shows the scheme of the nonconventional JFRTC. The FRT of function $f(x,y)$ with order α may be written as [22],

$$U(\xi,\eta) = \Im\alpha[f(x,y)] \tag{1}$$

In a fractional correlator, reference signal $u_r(x_0, y_0)$ and input signal $u(x_1, y_1)$ are fractional Fourier transformed with orders η_1 and η_2 respectively.

$$U_r(\xi,\eta) = K_1 \iint u_r(x_0,y_0) \times \exp\left[j\pi \frac{{x_0}^2 + {y_0}^2 + \xi^2 + \eta^2}{\tan\ \eta_1} - 2j\pi \frac{x_0 y_0 \xi\eta}{\sin\ \eta_1}\right] dx_0 dy_0 \tag{2}$$

$$U(\xi,\eta) = K_2 \iint u(x_1,y_1)] \times \exp\left[j\pi \frac{x_1^2 + y_1^2 + \xi^2 + \eta^2}{\tan\ \eta_2} - 2j\pi \frac{x_1 y_1 \xi\eta}{\sin\ \eta_2}\right] dx_1 dy_1 \tag{3}$$

Here (x_0,y_0) and (x_1,y_1) are spatial coordinates and (ξ, η) represent fractional domain coordinate. K_1 and K_2 are complex constants and η_1 and η_2 are order parameters. The joint fractional power spectrum (JFPS) is given by $J(\xi,\eta)$.

$$J(\xi,\eta) = U(\xi,\eta) \times U_r^*(\xi,\eta) \tag{4}$$

FRT of the JFPS $J(\xi,\eta)$ for order η_3 gives a sharp autocorrelation peak, if reference and input signals are identical [22]. The Correlation coefficient may be written as,

$$C(p,q) = K_3 \iint \left\{U(\xi,\eta) \times U_r^*(\xi,\eta)\right\} \times \exp\left[j\pi \frac{p^2 + q^2 + \xi^2 + \eta^2}{\tan\ \eta_3} - 2j\pi \frac{pq\xi\eta}{\sin\ \eta_3}\right] d\xi d\eta \tag{5}$$

K_3 is a complex constant. It can be expanded using Eqs. (2) and (3) as,

$$C(p,q) = K_1 K_2^* K_3 \iint \iint \iint \left\{U(x_1,y_1) \times U_r^*(x_0,y_0)\right\} \times \exp\left[-j\pi \frac{{x_0}^2 + {y_0}^2 + \xi^2 + \eta^2}{\tan\ \eta_1} + j\pi \frac{x_1^2 + y_1^2 + \xi^2 + \eta^2}{\tan\ \eta_2} + j\pi \frac{p^2 + q^2 + \xi^2 + \eta^2}{\tan\ \eta_3}\right] \times \exp\left[2j\pi \frac{x_0 y_0 \xi\eta}{\sin\ \alpha_1} - 2j\pi \frac{x_1 y_1 \xi\eta}{\sin\ \alpha_2} - 2j\pi \frac{\xi\eta pq}{\sin\ \eta_3}\right] dx_0 dy_0 dx_1 dy_1 d\xi d\eta \tag{6}$$

If the Eq. (7) is satisfied, the quadratic phase term with coordinates ξ and η in Eq. (6) is eliminated and integral with respect to ξ and η takes the form of Dirac integral [23–25].

$$-\frac{1}{\tan\ \eta_1} + \frac{1}{\tan\ \eta_2} + \frac{1}{\tan\ \eta_3} = 0 \tag{7}$$

The integral assumes maximum value, *i.e.* one if its argument becomes zero, otherwise it vanishes.

$$\iint \exp\left[2j\pi \frac{x_0 y_0 \xi\eta}{\sin\ \eta_1} - 2j\pi \frac{x_1 y_1 \xi\eta}{\sin\ \eta_2} - 2j\pi \frac{\xi\eta pq}{\sin\ \eta_3}\right] d\xi d\eta = \delta\left(\frac{x_0}{\sin^2\ \eta_1} - \frac{x_1}{\sin^2\ \eta_2} - \frac{p}{\sin^2\ \eta_3}\right) \delta\left(\frac{y_0}{\sin^2\ \eta_1} - \frac{y_1}{\sin^2\ \eta_2} - \frac{q}{\sin^2\ \eta_3}\right) \tag{8}$$

Therefore, correlation coefficient $C(p,q)$ attains its maximum value when the following condition is satisfied.

$$x_0 = \left(\frac{x_1}{\sin^2\ \eta_2} + \frac{p}{\sin^2\ \eta_3}\right) \sin^2\ \eta_1 \tag{9}$$

$$y_0 = \left(\frac{y_1}{\sin^2\ \eta_2} + \frac{q}{\sin^2\ \eta_3}\right) \sin^2\ \eta_1 \tag{10}$$

If we assume, $\eta_1 = \eta_2$ and $\eta_3 = \pm\pi/2$, so that Eq. (8) is satisfied, then Eqs. (9) and (10) become,

$$x_0 = x_1 + p \sin^2\ \eta_1 \tag{11}$$

$$y_0 = y_1 + q \sin^2\ \eta_1 \tag{12}$$

The intensity distribution of joint fractional correlation, is given by

$$C(p,q) = K_1 K_2^* K_3 \iint \{U(x_1,y_1) \times {U_r}^*(x_1 + p \sin^2\ \eta_2, y_1 + q \sin^2\ \eta_2)\} \times \exp\left[-j\pi \frac{x_1 p \sin^2\ \eta_2}{\tan\ \eta_2} - j\pi \frac{y_1 q \sin^2\ \eta_2}{\tan\ \eta_2}\right] \times dx_1 dy_1 \tag{13}$$

The correlation coefficient at the centre ($p = 0$ and $q = 0$) is given as,

$$C(0,0) = K_1 K_2^* K_3 \iint U(x_1,y_1)] \times U_r^*(x_1,y_1) dx_1 dy_1 \tag{14}$$