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Solitary wave solutions for perturbed nonlinear Schrodinger's equation with Kerr law nonlinearity under the DAM

M. Eslami *

Department of Mathematics, Faculty of Mathematical Sciences, University of Mazandaran, Babolsar, Iran

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ABSTRACT

In this present paper, we aim to extend the applications of direct algebraic method to solve a perturbed nonlinear Schrodinger's equation with Kerr law nonlinearity. It is shown that the proposed method is effective and general. Many different new complex solitary solutions are obtained. Some previous results are extended. These complex solitary wave solutions are expressed by hyperbolic function, trigonometric functions are rational functions.

1. Introduction

It is well known that the exact solutions of the perturbed nonlinear Schrodinger's equation with Kerr law nonlinearity has been extensively studied in the field of theoretical physics. The exact solitary wave solutions can be used to specify initial data for the incident waves in the numerical model and for the verification of the associated computed solution.

Recently many new approaches for finding the exact solutions to nonlinear wave equations have been proposed, such as, direct algebraic method [1], simplest equation method [2,3], tanh method [4,5], multiple exp-function method [6], Backlund transformation method [7], Hirotas direct method [8,9], transformed rational function method [10], and so on [11–16].

The direct algebraic method is a very powerful mathematical technique for finding exact solutions of nonlinear ordinary differential equations.

In this paper, we will consider the perturbed nonlinear Schrodinger's equation (NLSE) with Kerr law nonlinearity [17] with following form:

$$iu_t + u_{xx} + \alpha|u|^2u + i(\gamma_1 u_{xxx} + \gamma_2|u|^2u_x + \gamma_3(|u|^2)_x u) = 0, \tag{1}$$

where γ_1 is third order dispersion, γ_2 is the nonlinear dispersion, while γ_3 is also a version of nonlinear dispersion [18,19]. Eq. (1) describes the propagation of optical solitons in nonlinear optical fibers that exhibits a Kerr law nonlinearity. Eq. (1) has important application in various fields, such as semiconductor materials, optical fiber communications, plasma physics, fluid and solid mechanics.

This paper is organized as follows. In Section 2, we introduce the extended direct algebraic method briefly. In Section 3, we give many exact solutions of Eq. (1). In Section 4, a short conclusion will be given.

2. Extended direct algebraic method

For a given partial differential equation

$$G(u, u_x, u_t, u_{xx}, u_{tt}, \ldots) = 0, \tag{2}$$

* Corresponding author at: Department of Mathematics, Faculty of Mathematical Sciences, University of Mazandaran, Babolsar, Iran. Tel.: +98 9113251508.
E-mail address: meslami.edu@gmail.com

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We seek complex solutions of Eq. (2) as the following form:

$$u = U(\xi)e^{i(sx-\Omega t)}, \quad \xi = ik(x-ct), \tag{3}$$

where k and c are real constants. Under the transformation (3), Eq. (2) becomes an ordinary differential equation

$$\varphi(Ue^{i(sx-\Omega t)}, ikU'e^{i(sx-\Omega t)} + isUe^{i(sx-\Omega t)}, -ikcU'e^{i(sx-\Omega t)} - i\Omega Ue^{i(sx-\Omega t)}, \ldots) = 0. \tag{4}$$

where $u' = du/d\xi$. We assume that the solution of Eq. (4) is of the form

$$u(\xi) = \sum_{i=0}^{n} a_i F^i(\xi), \tag{5}$$

where $a_i (i = 1, 2, \ldots, n)$ are real constants to be determined later. $F(\xi)$ expresses the solution of the auxiliary ordinary differential equation

$$F'(\xi) = b + F^2(\xi), \tag{6}$$

Eq. (6) admits the following solutions:

$$\begin{aligned} &F(\xi) = \begin{cases} -\sqrt{-b}\tanh(\sqrt{-b}\xi), & b \prec 0 \\ -\sqrt{-b}\coth(\sqrt{-b}\xi), & b \prec 0 \end{cases} \\ &F(\xi) = \begin{cases} \sqrt{b}\tan(\sqrt{b}\xi), & b \succ 0 \\ -\sqrt{b}\cot(\sqrt{b}\xi), & b \succ 0 \end{cases} \\ &F(\xi) = -\frac{1}{\xi}, \quad b = 0 \end{aligned} \tag{7}$$

Integer n in (5) can be determined by considering homogeneous balance [3] between the nonlinear terms and the highest derivatives of $u(\xi)$ in Eq. (4). Now with substituting (5) into (4) with (6), then the left hand side of Eq. (4) is converted into a polynomial in $F(\xi)$, equating each coefficient of the polynomial to zero yields a set of algebraic equations for a_i, k, c. By solving the algebraic equations obtained in step 3, and substituting the results into (5), then we obtain the exact traveling wave solutions for Eq. (2).

3. Application to the perturbed nonlinear Schrodinger's equation with Kerr law nonlinearity

Next, we study Eq. (1). Considering the following complex transformation:

$$u(x,t) = U(\xi)e^{i(sx-\Omega t)}, \quad \xi = ik(x-ct), \tag{8}$$

where s, Ω, k and c are constants, all of them are to be determined. So

$$\begin{aligned} u_t &= -i(\Omega U + kcU_\xi)e^{i(sx-\Omega t)}, \\ u_x &= i(sU + kU_\xi)e^{i(sx-\Omega t)}, \\ u_{xx} &= -(s^2U + 2ksU_\xi + k^2U_{\xi\xi})e^{i(sx-\Omega t)}, \\ u_{xx} &= -i(s^3U + 3ks^2U_\xi + 3k^2sU_{\xi\xi} + k^3U_{\xi\xi\xi})e^{i(sx-\Omega t)}. \end{aligned} \tag{9}$$

Substituting (8) and (9) into Eq. (1), we have

$$\gamma_1 k^3 U_{\xi\xi\xi} + (kc - 2ks + 3ks^2\gamma_1)U_\xi + (-k^2 + 3\gamma_1 k^2 s)U_{\xi\xi} - (k\gamma_2 + 2k\gamma_3)U^2U_\xi + (\alpha - \gamma_2 s)U^3 + (\Omega - s^2 + \gamma_1 s^3)U = 0 \tag{10}$$

For the solutions of Eq. (10), with the aid of direct algebraic method we make the following ansatz

$$U(\xi) = \sum_{i=0}^{n} a_i F^i(\xi),$$

where a_i are all real constants to be determined, n is a positive integer which can be determined by balancing the highest order derivative term with the highest order nonlinear term. Balancing $U_{\xi\xi\xi}$ with U^2U_ξ then gives $n + 3 = 2n + n + 1 \Rightarrow n = 1$. Therefore, we may choose

$$U(\xi) = a_1 F + a_0, \tag{11}$$

Substituting (11) along with (6) in Eq. (9) and then setting the coefficients of F^i, $(i = 1, 2, 3, \ldots)$ to zero in the resultant expression, we obtain a set of algebraic equations involving a_0, a_1, a, b and s, Ω, k as

$$\begin{aligned} &6a_1\gamma_1 k^3 - a_1^3(k\gamma_2 + 2k\gamma_3) = 0, \\ &2a_1(-k^2 + 3\gamma_1 sk^2) - (k\gamma_2 + 2k\gamma_3)a_1^3 + (\alpha - \gamma_2 s)a_1^3 = 0, \\ &8(kc - 2ks + 3\gamma_1 s^2 k)a_1 b + (kc - 2ks + 3\gamma_1 s^2 k)a_1 - (k\gamma_2 + 2k\gamma_3)(a_1^3 b + a_1 a_0^2) + 3(\alpha - \gamma_2 s)a_1^2 a_0 = 0, \\ &2ba_1(-k^2 + 3\gamma_1 sk^2) - 2a_1 a_0^2 b(k\gamma_2 + 2k\gamma_3) + 3a_1 a_0^2(\alpha - \gamma_2 s) + a_1(\Omega - s^2 + \gamma_1 s^3) = 0, \\ &2a_1 b^2\gamma_1 k^3 + a_1 b(kc - 2ks + 3\gamma_1 s^2 k) - a_1 a_0^2 b(k\gamma_2 + 2k\gamma_3) + a_0^3(\alpha - \gamma_2 s) + a_0(\Omega - s^2 + \gamma_1 s^3) = 0. \end{aligned}$$