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Dispersion properties of lossy, dispersive, and anisotropic left-handed material slab waveguide

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ABSTRACT

An asymmetric three-layer slab waveguide with air as a substrate, lossless dielectric medium as a guiding layer, and lossy, dispersive, and anisotropic left-handed material (LHM) as a cladding was investigated. The LHM cladding layer was assumed to exhibit positive permittivity and permeability in the transverse directions and negative ones along the longitudinal direction (z-axis). The dispersion equations were derived and corresponding dispersion curves were plotted and studied for TE and TM polarized light. The dispersion properties were studied with different parameters of the proposed structure. A set of interesting features were observed such as the existence of the fundamental mode in a narrow frequency range, positive group velocity which becomes faster as the frequency increases, and crucial dependence of the dispersion curves on the anisotropy of the LHM cladding.

1. Introduction

Left-handed material (LHM) in which the electric permittivity ε and the magnetic permeability μ are simultaneously negative was proposed by Veselago in 1968 [1]. He predicted a number of peculiar properties that differ from normal materials, such as negative index of refraction and reversed Cerenkov radiation and Doppler effect. The possibility of fabricating LHMs by some novel artificial microstructured metallic materials was proposed by Pendry et al. [2,3]. In 2001, the first LHM was fabricated over the microwave frequency by Smith et al. [4]. Since then slab waveguide structures comprising LHMs have become an interesting topic [5–25]. Theoretical investigation of the electromagnetic properties of multi-layer LHM waveguide was investigated [5–7]. A unique negative lateral shift was demonstrated for a Gaussian beam reflected from a grounded LHM slab, which is different from the conventional shift caused by a regular grounded slab [8]. The guided and surface waves propagating along a slab waveguide structure with a LHM clad or substrate have been presented using three normalized parameters [9–12]. The existence of surface waves for which the wave number is purely imaginary was reported in LHM waveguides [13,14]. The enhancement of photon tunneling by a slab of anisotropic LHM material was found [15]. The dispersion properties and energy flux properties in a LHM slab waveguide was obtained by Pan et al. in 2009 [16]. The electromagnetic guided waves in symmetric LHM waveguide have been investigated [17]. It was shown theoretically that under some conditions the amplitude of the evanescent wave would be amplified exponentially when it is transmitted through a slab of uniaxially anisotropic LHM [18]. The dispersion properties of guided modes in a metal-clad waveguide comprising a LHM guiding layer have also been studied [19]. Young et al. have investigated the guidance characteristics of circular LHM rod waveguide including the dispersion properties and power confinement characteristics [20]. Nonlinear guided waves in a LHM waveguide surrounded by a Kerr-like nonlinear dielectric have been analyzed [21]. It is found that such a waveguide can support fast and slow symmetric and antisymmetric nonlinear modes. Surface and guided modes by a single interface and a slab waveguide containing LHMs were analyzed by using a ray picture [22]. The transfer function of the discretized perfect lens in finite-difference time-domain and transfer matrix method simulations was investigated [23]. LHM waveguides have been proposed for many optoelectronics applications. Among these applications of LHM waveguide structures is slab waveguide sensing [24] in which minute changes in the index of refraction of an aqueous cladding can be detected with high sensitivity.

In this paper, the dispersion properties of electromagnetic waves guided by an asymmetric three-layer waveguide are discussed. The waveguide structure under consideration comprises an air substrate, a dielectric guiding film, and an LHM cladding. The LHM is assumed to dispersive, lossy, and anisotropic medium. Both TE and TM polarizations are treated.

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2. Dispersion relations

Fig. 1 shows an asymmetric three-layer waveguide structure. The core layer is assumed to be lossless dielectric of thickness h, permittivity ε_2, and permeability μ_2. The substrate is considered to be air with permittivity ($\varepsilon_1 = 1$) and permeability ($\mu_1 = 1$). The cladding layer is anisotropic LHM with permittivity tensor $\leftrightarrow \varepsilon_3(\varepsilon_{3x}, \varepsilon_{3y}, \varepsilon_{3z})$ and permeability tensor $\leftrightarrow \mu_3(\mu_{3x}, \mu_{3y}, \mu_{3z})$. Taking the transverse electric (TE) mode into consideration, the electric field is polarized along the y-axis. When ω and β denote angular frequency and longitudinal propagation constant, the electric field can be written as $E_y(x, z, t) = E(x)\exp[-i(\omega t - \beta z)]$. From Maxwell's equations, the electromagnetic fields in the anisotropic cladding satisfy the relations

$$-i\beta H_x - \frac{\partial H_x}{\partial x} = i\omega\varepsilon_{3y}E_y, \tag{1}$$

$$H_x = -\frac{\beta}{\omega\mu_{3x}}E_y, \tag{2}$$

$$H_z = \frac{i}{\omega\mu_{3z}}\frac{\partial E_y}{\partial x}. \tag{3}$$

In the substrate and guiding layers, the field equations are similar to Eqs. (1)–(3) with ε_i and μ_i of the ith layer replace the tensor elements appearing in these equations. The electric fields of TE modes in the three regions can be written as

$$E_1 = A_1\exp(k_1 x), \quad x < 0, \tag{4}$$

$$E_2 = A_2\cos(k_2 x - \varphi), \quad 0 < x < h, \tag{5}$$

$$E_3 = A_3\exp[-k_3(x - h)], \quad x > h, \tag{6}$$

where A_1, A_2, and A_3 are electric field amplitudes in each region. $k_1 = \sqrt{\beta^2 - k_0^2\varepsilon_1\mu_1}$ and $k_3 = \sqrt{(\mu_{3z}/\mu_{3x})(\beta^2 - k_0^2\varepsilon_{3y}\mu_{3x})}$ are the evanescent rates in the substrate and cladding, respectively. $k_2 = \sqrt{k_0^2\varepsilon_2\mu_2 - \beta^2}$ is the transverse wave number in the guiding region which can either be real for guided modes or imaginary for surface waves. k_0 is the vacuum wave number and $\varphi = \tan^{-1}(k_1\mu_2/k_2\mu_1)$ is the phase shift of guided mode in region 2. The boundary conditions can be applied at $x = 0$ and $x = h$. The dispersion relation of TE guided modes can be obtained as follow

$$k_2 h = \tan^{-1}\left(\frac{\mu_2 k_3}{\mu_{3z}k_2}\right) + \tan^{-1}\left(\frac{\mu_2 k_1}{\mu_1 k_2}\right) + m\pi, \tag{7}$$

where m is the order of the guided mode.

For TM oscillating guided modes, the dispersion relation is given by Eq. (7) with the permeability is replaced by the corresponding permittivity. The evanescent rate in the cladding region is written as $k_3' = \sqrt{(\varepsilon_{3z}/\varepsilon_{3x})(\beta^2 - k_0^2\mu_{3y}\varepsilon_{3x})}$ for TM modes.

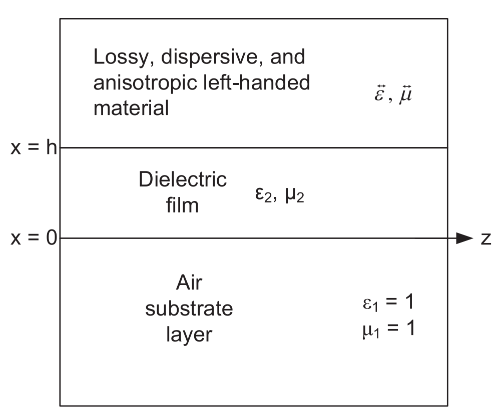

Fig. 1. Schematic geometry of a three-layer slab waveguide structure including lossy, dispersive, and anisotropic left-handed material as a cladding layer.

The realization of left-handed media with negative index of refraction is performed by combining periodic arrays of metallic wires and split-ring resonators or employing photonic crystals. Considering the LHM cladding region to be a combination of periodic arrays of metallic wires and split-ring resonators, the material inherent properties such as dispersion and anisotropy must be taken into account. The LHM cladding is assumed to have positive permittivity and permeability in the transverse directions and negative ones along the longitudinal direction, then $\varepsilon_{3x} > 0$, $\varepsilon_{3y} > 0$, and $\varepsilon_{3z} = \varepsilon(\omega) < 0$; and $\mu_{3x} > 0$, $\mu_{3y} > 0$, and $\mu_{3z} = \mu(\omega) < 0$. The experimental models for $\varepsilon(\omega)$ and $\mu(\omega)$ in the microwave range are given by

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}, \tag{8}$$

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\gamma\omega}, \tag{9}$$

where ω_p is the plasma frequency, ω_o is the resonance frequency, γ is the electron scattering rate, and F is the fractional area of the unit cell occupied by the split ring.

3. Numerical results

Fig. 1 shows the geometry of the slab waveguide under consideration. The cladding region is assumed to be filled with a dispersive anisotropic LHM. Uniaxial anisotropy is assumed in which $\varepsilon_{3x} = \varepsilon_{3y}$ and $\mu_{3x} = \mu_{3y}$ with the considerations $\varepsilon_{3x} \neq \varepsilon_{3z}$ and $\mu_{3x} \neq \mu_{3z}$. The permittivity tensor is then given by $\leftrightarrow \varepsilon_3(\varepsilon_{3x}, \varepsilon_{3x}, \varepsilon_{3z})$ and the permeability tensor becomes $\leftrightarrow \mu_3(\mu_{3x}, \mu_{3x}, \mu_{3z})$. The LHM is assumed to exhibit positive electric permittivity and magnetic permeability in the transverse directions with $\varepsilon_{3x} = \varepsilon_{3y} = 2.25$ and $\mu_{3x} = \mu_{3y} = 1$. The longitudinal elements (ε_{3z} and μ_{3z}) are assumed to be negative and obey the experimental model given by Eqs. (8) and (9). For $F = 0.56$, $\omega_0 = 4.0$ GHz, and $\omega_p = 10.0$ GHz, the frequency range $4.0 < \omega < 6.0$ in GHz provides negative values of ε_{3z} and μ_{3z}. The LHM cladding is first assumed to lossless with $\gamma = 0$. The parameters of the guiding layer and substrate are assumed to $\varepsilon_2 = 4$, $\mu_2 = 1$, $\varepsilon_1 = 1$, and $\mu_1 = 1$. Based on the above parameters, the dispersion relation can be solved numerically for the longitudinal propagation constant β which is given by $\beta = k_0 N$, with N is the effective refractive index of the guided mode. The dispersion properties can be studied from the dispersion curves which show the dependence of the effective refractive index on the frequency of the propagating wave. Fig. 2 shows the dispersion curves of the fundamental TE oscillating guided mode for different

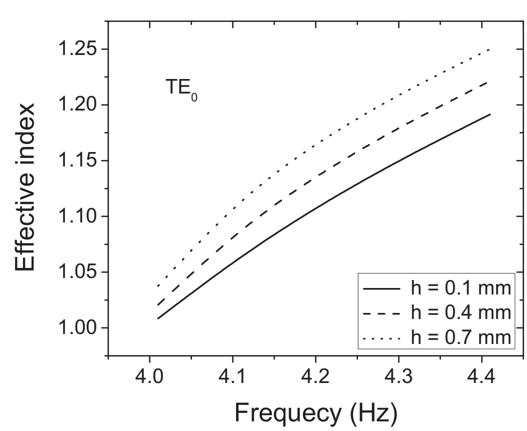

Fig. 2. Dispersion curves of oscillating fundamental mode (TE_0) for different guiding layer thicknesses for $\varepsilon_1 = 1$, $\mu_1 = 1$, $\varepsilon_2 = 4$, $\mu_2 = 1$, $F = 0.56$, $\omega_0 = 4.0$ GHz, $\omega_p = 10.0$ GHz, $\gamma = 0$, $\varepsilon_{3x} = \varepsilon_{3y} = 2.25$, and $\mu_{3x} = \mu_{3y} = 1$.