Optik 125 (2014) 5566–5572

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Variance squeezing and information entropy squeezing via Bloch coherent states in two-level nonlinear spin models

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ARTICLE INFO

Article history:
Received 21 October 2013
Accepted 15 May 2014

PACS:
42.50.Ct
42.50.Md
42.50.Dv
03.65.Yz

Keywords:
Variance squeezing
Information entropy
Bloch coherent states
Atomic population inversion
Quantum state purity

ABSTRACT

The nonclassical squeezing effect emerging from a nonlinear coupling model (generalized Jaynes–Cummings model) of a two-level atom interacting resonantly with a bimodal cavity field via two-photon transitions is investigated in the rotating wave approximation. Various Bloch coherent initial states (rotated states) for the atomic system are assumed, i.e., (i) ground state, (ii) excited state, and (iii) linear superposition of both states. Initially, the atomic system and the field are in a disentangled state, where the field modes are in Glauber coherent states via Poisson distribution. The model is numerically tested against simulations of time evolution of the based Heisenberg uncertainty relation variance and Shannon information entropy squeezing factors. The quantum state purity is computed for the three possible initial states and used as a criterion to get information about the entanglement of the components of the system. Analytical expression of the total density operator matrix elements at $t>0$ shows, in fact, the present nonlinear model to be strongly entangled, where each of the definite initial Bloch coherent states is reduced to statistical mixtures. Thus, the present model does not preserve the modulus of the Bloch vector.

1. Introduction

The Jaynes–Cummings model (JCM) was originally proposed for describing the spontaneous emission in a semi-classical manner [1–4]. This model consists of a single two-level atom (qubit) and a single cavity mode of the electromagnetic field. The JCM interaction between the atom and the cavity mode is obtained by the rotating wave approximation (RWA), so that each photon creation causes an atomic de-excitation and each photon annihilation causes an atomic excitation. The JCM is an analytically tractable quantum mechanical model. Moreover, it is simple enough for expressing the basic and most important characteristics of the matter–radiation interaction.

In the atom–field interaction scenario, where the atom is initially prepared in the ground or excited state and the cavity mode in the coherent state, the level structure of the atom leads to the prediction of a wide range of experimentally verifiable coherent phenomena. Probably the most notable among them is the observation of the periodic spontaneous collapse and the revival of the Rabi oscillations during the time-evolution [5–9], a phenomenon that was experimentally demonstrated in the 1980s [10]. This phenomenon can be regarded as a direct evidence for discreteness of energy states of photons and clearly is a manifestation of the role of quantum mechanics in the coherence and fluctuation properties of radiation–matter systems. Particularly relevant is the atomic squeezing phenomenon. It reflects the nonclassical behavior for the quantum systems and is one of the most interesting phenomena in the field of quantum optics. In fact, the spin squeezing can be considered as a quantum strategy [11] which aims at redistributing the fluctuations of two orthogonal spin directions between each other. It was theoretically shown that spin squeezed states are useful quantum resources to enhance the precision of atom interferometers [12] and the connection between spin squeezing and entanglement was pointed out [13]. Thus, the JCM has a fully quantum property, which cannot be explained by semi-classical physics.

Because of its distinct advantages the extensions of the basic JCM have been extremely plentiful, e.g., the generalized N-level JCM [14,15], JCM beyond RWA [16–19], non-linear JCM [8,9,20,21], and JCM with intensity-dependent coupling [19,22–24].

In the present paper, the so far uninvestigated non-dissipative generalized JCM using Bloch coherent states [25] in the initial

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http://dx.doi.org/10.1016/j.ijleo.2014.07.065
0030-4026/

atomic state will be explored through numerical simulations of time evolution of the based Heisenberg uncertainty relation variance (which is regarded as the standard limitation on measurements of quantum fluctuations), and Shannon information entropy squeezing factors.

Spin or angular momentum systems have often been regarded as squeezed if the uncertainty of one spin component, say $\langle \Delta S_x^2 \rangle$ or $\langle \Delta S_y^2 \rangle$, is smaller than $1/2|\langle S_z \rangle|$. This definition implies that a coherent spin state is already squeezed if it is placed in an appropriate system of coordinates, and also that spin can be squeezed by just rotating the coherent spin state. In spin systems, the squeezing occurs on the phase sphere (spherical phase space). Unlike boson squeezing, the quasiprobability distribution cannot be homogeneously or globally squeezed in one direction over the whole phase space. If a spin component is shrunk around a certain point on the sphere, it must be stretched around another point. This imposes a fundamental restriction on the reduction in quantum noise.

As an alternative to the Heisenberg uncertainty relation the quantum uncertainty can also be studied by using quantum entropy theory which can overcome the limitations of the Heisenberg uncertainty relation [26]. In this paper, we find that the entropic uncertainty relation can be used as a general criterion for the squeezing of a two-level atomic system.

The use of Bloch coherent states (rotated states) is a novel feature of the present model since it offers the possibility of considering various initial atomic states. In fact, an appropriate choice of the rotation angle θ leads to different initial states on the Bloch sphere, namely, (a) ground state ($\theta=\pi$); (b) excited state ($\theta=0$); and (c) a linear superposition of both states ($0<\theta<\pi$). It will be assumed that two degenerate modes (i.e., $\nu_1=\nu_2$) of the electromagnetic field and two-photon transitions are involved in the resulting highly non-linear Hamiltonian and that the spin transition frequency ω is nearly resonant with the two (degenerate) modes, i.e., $\Delta_j=\omega-\nu_j=0$ ($j=1, 2$), where Δ_j is the detuning parameter for the mode j. The atom and the cavity modes are initially in thermal equilibrium at a certain finite low temperature $\beta(=1/kT)$. Thus, we can describe an initial probability distribution of quantum states of the system with the canonical ensemble. Moreover, we assume that the time-evolution of the system is governed by a unitary operator generated with the JCM Hamiltonian. This implies that the system does not suffer from dissipation and its time-evolution is reversible.

The remainder of the paper is organized as follows. Section 2 describes the initial conditions and introduces the interaction picture representation of the model, leading to a time evolution operator from which the dynamics of the atomic system–field interaction is examined. Section 3 discusses the numerical simulations. The quantum *purity* is computed for the three possible initial states and used as a criterion to get information about the entanglement of the components of the system. The second-order statistical moments necessary to compute the Heisenberg uncertainty relation variance and the information entropy squeezing factors are computed in the rotated basis for the three above mentioned possible initial atomic states. The time-evolution of the atomic population inversion is also computed. The paper ends up with conclusions in Section 4.

2. Theoretical background

2.1. Bloch sates and initial conditions

Let us consider a bosonic system S, with Hilbert space $\mathcal{H}^{(S)}$ which is coupled with a two-level atom ($S=1/2$), with Hilbert space $\mathcal{H}^{(B)}$. It is assumed that the complete system is in thermal equilibrium with a reservoir at temperature β^{-1}. It is important to keep in mind that the presence of the reservoir only takes the two-level atom and the bosonic modes in thermal equilibrium. Let us denote by $\mathcal{H}_S$, $\mathcal{H}_B$, and $\mathcal{H}_I$ the Hamiltonians of the bosonic field, the two-level atom, and the interaction between both systems, respectively. The Hamiltonian for the total system can be written as

$$\mathcal{H}=\mathcal{H}_S\otimes I_B+I_S\otimes\mathcal{H}_B+\mathcal{H}_I\equiv\mathcal{H}_0+\mathcal{H}_I, \tag{1}$$

where I_S and I_B denote the identities in the Hilbert spaces of the bosonic field and the two-level atom. Thus, under the RWA the Hamiltonian of a two-level atom interacting resonantly with a bimodal cavity via two-photon transitions becomes (a system of units in which $\hbar$=1 is used throughout the paper)

$$\mathcal{H}=\underbrace{\sum_{j=1}^{2}\nu_j a_j^\dagger a_j\otimes I_B+\omega I_S\otimes S_z}_{\mathcal{H}_0}+\underbrace{\sum_{j=1}^{2}G_j(\sigma_+\otimes a_j^2+\sigma_-\otimes a_j^{\dagger 2})}_{\mathcal{H}_I}, \tag{2}$$

where G_j is the atom–field coupling constant (vacuum Rabi frequency) for the mode j and $a_j^\dagger$ (a_j) is the associated canonical creation (annihilation) bosonic operator.

The connection of spins to interferometry becomes obvious when realizing that an interferometer is essentially a two-level system with states $|a\rangle$ and $|b\rangle$. Each particle in the interferometer can be regarded as an elementary spin, $\sigma=1/2$, corresponding to the two interferometer states. Thus, we start by describing the single-qubit computational basis representing the ground and the excited state of the atom as two-components vectors

$$|a\rangle=\begin{pmatrix}0\\1\end{pmatrix}\quad |b\rangle=\begin{pmatrix}1\\0\end{pmatrix}, \tag{3}$$

and therefore the Pauli atom-flip operators in Eq. (2) are given by

$$\sigma_+=|b\rangle\langle a|\quad \sigma_-=|a\rangle\langle b|. \tag{4}$$

The Hilbert space of the atomic operators is spanned by the Dicke states, which are simply the usual spin angular momentum states $|SM\rangle$ ($M=-S, -S+1$) obtained as the simultaneous eigenstates of the $SU(2)$ Casimir operators S^2 and $S_z=1/2[S_+, S_-]$, where $S_\pm$ are the raising and lowering spin operators, i.e., $S_\pm=1/2\sigma_\pm$. The Dicke states are then given by

$$|SM\rangle=\frac{1}{(M+S)!}\begin{pmatrix}2S\\M+S\end{pmatrix}^{-1/2}S_+^{M+S}|S-S\rangle, \tag{5}$$

with eigenvalue M and where the ground state $|S-S\rangle$ is defined by $S_-|S-S\rangle$=0. Let us consider the rotation operator $R_{\theta,\phi}$ which produces a rotation through the coherence angle θ about an axis $\hat{n}=(\sin\phi, -\cos\phi, 0)$

$$R_{\theta,\phi}=e^{-i\theta S_n}=e^{-i\theta(S_x\sin\phi-S_y\cos\phi)}=e^{\vartheta S_+-\vartheta^* S_-}, \tag{6}$$

where

$$\vartheta=\frac{1}{2}\theta e^{-i\phi}\quad (0\le\theta\le\pi; 0\le\phi\le 2\pi). \tag{7}$$

A coherent atomic state, or Bloch state, $|\theta, \phi\rangle$ is obtained by rotation of the ground state $|S-S\rangle$, i.e.,

$$|\theta,\phi\rangle\equiv R_{\theta,\phi}|S-S\rangle, \tag{8}$$

which is the group definition of the atomic coherent states. The Bloch spin coherent states $|\theta, \phi\rangle$ satisfy a completeness relation given by

$$(2S+1)\int|\theta,\phi\rangle\frac{d\Omega}{4\pi}\langle\theta,\phi|=1, \tag{9}$$

where $d\Omega=\sin\theta\, d\theta\, d\phi$ is the solid-angle volume element at (θ, ϕ) on $\mathcal{S}^2$ (Bloch sphere). The Bloch sphere is a well-known tool in