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Split Bregman iteration solution for sparse optimization in image restoration

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ABSTRACT

It is always a challenging task to develop effective and accurate models for robust image restoration. In this paper, the family of sparse and redundant representation frameworks is considered as an alternative for the above problem. The principle of the family is expatiated on the development and research progress. Two well-known denoising methods are presented and analyzed on their properties. The K-SVD algorithm is an effective method for sparse representation. The iteratively approximate algorithms are always used for the solution of sparse coding operations. Here, a convexification of the l_0 norm to the l_1 norm is adopted in the implementation of K-SVD method. Then a split Bregman iteration solution is proposed for l_1 regularization problems in the performance of the sparse representation of the K-SVD algorithm. The split Bregman iterative method is well studied and fused into the famous K-SVD method. The *PSNR* (Peak Signal to Noise Ratio) and *MSSIM* (Mean Structural Similarity) are used to evaluate the performance of those methods. Experimental results on different types of images indicate that our proposed method not only achieve comparable results with the state of art methods, but also make the original method more efficient. Besides, it also provides a valuable and promising reference for image restoration techniques.

1. Introduction

Traditionally, it is known that removing noises or other degradation from signals is possible only if some prior information is available. The models of degradation sources (noise and blurring) that we usually assumed are additive, multiplicative, convolution, including Gaussian model, Poisson model, and so forth. Then some procedures are designed and implemented to remove or reduce the effects caused by above degradation. However, some statistical models are always not acquired for complex signals, such as natural images. Image restoration is a fundamental and crucial research point in image processing with numerous literatures published every year. Excellent restoration algorithms not only attenuate random noises efficiently, but also can retain important edge information. Practically, noises are always randomly distributed. It is not accurately to characterize them that only one model is defined. The noises cannot be removed completely. Various denoising approaches have been tried to restore the original signals approximately to the best of our ability. In frequency domain, the ultimate reason for the limitation of conventional restoration methods is the overlaps of frequency bands between useful signals and noises. However, the survey on various image restoration methods is will not been taken in this paper. Instead, we concentrate on a specific field of restoration methods using sparse and redundant representation, which have been dominated to be quite effective and achieved the state of art experiment results.

Recent years, redundant and sparse representations of images have achieved great success in image processing field [1,2]. Efficient dictionary learning methods are generated based on it. If the sampling of a signal has only finite nonzeros, the rest of all other samplings are zero or approximate to zero. The signal is referred to have sparsity. In reality, the natural images that we acquired by imaging sensors always do not have the property of sparsity, or in stringent speaking, the number of natural images satisfies the qualification of sparsity is few. Although some samplings are small, they cannot be omitted for that they do not equal or approximate to zero. So we have to use the concept of compressible signal defined by Davenport [3]. Compressible signal means that the signals may become sparse in some transformed form. Sparsity representation can be used in compression, statistics, learning theory and reconstruction tasks for it has some properties as follows [3]. First, sparse decomposition do not have to need the prior knowledge of images and statistics property of the noises. The signal can be decomposed based on over-complete. The set can be chosen flexibly and

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adaptively by the characteristics of image signal itself. Besides, the results of the decomposition will be a very simple representation. Second, the relationship between original image signals and measurements is incoherence. There are some inherent relations among the elements of real image signals. Simultaneously, the distributions of noises are always independent, random, discrete. Third, the image signals can be decomposed using characteristics of atoms in dictionary. The image is represented by the atoms of over-complete set sparsely. It is known that the solution of l_0 norm problem has been demonstrated as NP hard. The general methods used are mostly based on some greedy algorithms, such as matching pursuit (MP) or orthogonal matching pursuit (OMP) algorithm, to compute iteratively and exact useful information. The convex relaxation of l_0 norm has been proposed as an efficient scheme. Here we proposed the split Bregman iteration solution [4] to solve the l_1 regularized problems in image restoration.

The above sparse optimization issue has been studied from different perspective by scientists and engineers widely with lots of literatures [1,5–9] published in recent years. On the understanding and analysis by predecessor, this paper clarifies the problem in another point of view: the sparse optimization of l_1 regularization problems. First, this paper attempted to make a introduction of the development of sparse representation research and the well-known method: K-SVD, proposed by Aharon [7]. The split Bregman iteration solution is proposed to use for the well-known l_1 regularization problem in dictionary learning. Additionally, the differences and relationships are compared and analyzed between proposed method and the original algorithms. We compared with some other state of the art denoising methods, such as kernel regression [6], bilateral filtering [10], K-SVD [7], BM3D [5], CSR [11], and some other improved K-SVD algorithm. Finally, we presented our thinking about disadvantages and the future development direction of sparse representation method.

This paper is organized as remainder: in Section 2, we show the developments and principle of image restoration model in sparse and redundant representation framework. Section 3 gives the solution of the Bregman iteration solution to this problem. Section 4 shows the design of experiments and the experimental analysis of proposed method is discussed. Finally, gives a conclusion about the study is given in Section 5.

2. Background and prior art

There are some differences from conventional image restoration methods and the method based on sparse representation. The latter extracts sparse component from the original image and utilizes the sparse components to reconstruct original image. Then the denoised image is derived without any degradation. The observation model is

$$\mathbf{y} = \mathbf{x} + \mathbf{n} \tag{1}$$

The purpose of image restoration algorithms is to remove the degradative components $\mathbf{n}$. The model of noise degradations maybe in zero mean independent identically distributed additive noise, Poisson, or no particular statistics properties form. They are introduced in the process of measurement of signal $\mathbf{y}$. $\mathbf{x}$ is the clean original image.

These years, many researchers have paid a lot of attention to redundant representation for the shift invariance property. The content of image is treated equally no matter it is edges or not in processing. Until recently, some methods on nonlocal sparse model have achieved impressive performances, including nonlocal mean filtering (NLM) [12], Gaussian scale mixtures (GSM) [13], block matching with 3D filtering (BM3D) [5], and so forth.

When dealing with restoration problems based on sparse and redundant representation, the well-known optimization problem and its variant have to be treated.

$$\min_x ||\mathbf{x}||_0 \text{ subject to } ||\mathbf{Ax} - \mathbf{y}||_2^2 \le \varepsilon \tag{2}$$

$$\min_x ||\mathbf{y} - \mathbf{Ax}||_2 \text{ subject to } ||\mathbf{x}||_0 \le \varepsilon \tag{3}$$

where Eq. (2) is the well-known BPDN (Basis Pursuit Denoising) problem. While Eq. (3) is the celebrated LASSO (Least Absolute Shrinkage and Selection Operator) problem. The value of threshold ε is the error tolerance, which dependents on the standard deviation of noises. $\mathbf{Ax}$ is the estimate of the clean original image, $\mathbf{A}$ is the measurement matrix. $||*||_0$ is the l_0 norm, which is the number of nonzero entries of a matrix.

The well-known adaptive sparse representation framework is K-SVD [7]. It is a generalization and extension edition of K-means algorithm. The intention of the method is as follows. The measurement matrix $\mathbf{A}$ is obtained through learning from the contaminated image $\mathbf{y}$. The basis function of the algorithm can not only represents the image concisely, but also can removes noises efficaciously through reconstruction algorithms. The first step of the algorithm is partitioning the image into overlapped regions with size $\sqrt{m} \times \sqrt{m}$. The adaptive dictionary $\mathbf{A}$ is learned from these partitioned regions. Then the sparse estimations are made on every region based on the learned dictionary $\mathbf{A}$. The final restoration image was obtained by averaging the sparse estimation of the image region. The authors [7] extended the celebrated clustering algorithm K-means into sparse representation and applied it in image restoration field.

The objective function of the K-SVD is minimized as the following:

$$\min_{D,X}\{||\mathbf{Y} - \mathbf{DX}||_F^2\} \quad \text{s.t.} \forall i, \quad ||x_i||_0 \le T_0 \tag{4}$$

where $\mathbf{Y}$ is the set of all image signals, for $\{i = 1, \ldots, n\}$. $\mathbf{D}$ denotes the elements of dictionary. $\mathbf{X}$ is the sparse representation coefficients composed by x_i. $||L||_F$ is the Frobenius norm, $||L||_F = \sqrt{\sum_{ij} L_{ij}^2}$. After the update of the dictionary coefficients, the $\mathbf{D}$ and $\mathbf{X}$ are proposed to be fixed. There is only one column d_k of $\mathbf{D}$ updated once. The corresponding coefficient of kth row in $\mathbf{X}$ is x^k. The updating mechanism of d_k can be written as below:

$$||\mathbf{Y} - \mathbf{DX}||_F^2 = \left\| (\mathbf{Y} - \sum_{i \ne k} d_i x^i) - d_k x^k \right\|_F^2 = ||E_k - d_k x^k||_F^2 \tag{5}$$

where $\boldsymbol{E}_k$ is the rough error estimation of all samples except the kth atom. The singular value decomposition (SVD) is applied to the decomposition of $\boldsymbol{E}_k = U \Delta V^{\mathrm{T}}$. The first column of U and V are replaced by d^k and x^k correspondingly. The other dictionary elements of $\mathbf{D}$ are updated using the same method iteratively.

After the K-SVD proposed, there are various improved editions presented. In paper [14], Mazhar proposed a technique called Enhanced K-SVD (EK-SVD) algorithm. It prunes the underutilized or shared similarity elements to produce a well trained dictionary, which can get rid of the redundant elements. In paper [15], Rubinstein proposed a sparse dictionary based on a sparsity model of the dictionary atoms from a base dictionary. They referred to the work as sparse K-SVD algorithm, and used it in medical image restoration. They also in paper [16], Zhang extended the K-SVD algorithm by considering the representation error in the objective function. This change makes the algorithm having the information for discrimination. The discriminative K-SVD (DK-SVD) algorithm was successfully applied in face recognition and achieved very impressive experimental performances. Recently, Dong [11] proposed an novel framework that unified the sparsity and clustering