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Off-axial contribution of beam self-focusing in plasma with density ripple

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ABSTRACT

Off-axial contribution of beam self-focusing in plasma with density ripple is investigated. Apply paraxial ray theory and Wentzel–Krammers–Brillouin approximation, the results shown that, in interaction of laser and plasma with density ripple, beam self-focusing presents some interesting diverse features when off-axial contribution is obvious. In the paper, we find, on the one hand, density ripple can minimize the defocusing and beam still retains a localized profile with an oscillatory self-focusing and defocusing, on the other hand, with the increase of off-axial contribution, laser beams presents four various self-focusing features, which laser beam intensity profile splits into three-splitted with central axial convex profile, three-splitted with equal amplitude profile, three-splitted with central axial concave profile and two-splitted intensity profile.

1. Introduction

There has been widely interest in interaction of laser-plasma on account of its relevance to laser fusion [1], particle acceleration [2] and other applications [3]. Many nonlinear phenomenon has been studied widely theoretically and experimentally [4–6], such as, self-focusing [7], filamentation [8] and stimulated Raman scattering [9] etc. Beam self-focusing has been widely studied as a crucial nonlinear phenomenon, due to effect on the laser propagation [10], the coupling rate of laser-plasma [11] and other nonlinear processes [12]. The present physical mechanism of beams self-focusing mainly includes ponderomotive force [13], collision nonlinearity [14] and relativistic nonlinearity [15].

In laser-plasma interaction, the pulse may acquire a minimum spot size due to the self-focusing, the hot spot have been observed by lots experiments, including ponderomotive, collision nonlinearity and relativistic nonlinearity results in light spot [16]. Self-focusing was firstly predicted by Askaryan in 1962 [17], which has been widely researched by applied various method in some nonlinear media even since, such as, moment theory approach [18], variational approach [19], source-dependent expansion method [20] and paraxial ray theory (PR) [21]. Due to relatively distinct physical process and fairly simple calculation, the PR theory has been widely in self-focusing. Akhmanov et al. [22] firstly applied PR theory and Wentzel–Krammers–Brillouin (WKB) approximation to study self-focusing. Sodha et al. [23,24] and Kaw et al. [25] developed the analytical approach and further research self-focusing in laser-plasma interaction. PR theory is based on the expansion of the eikonal S and irradiance A_0^2 up to square term r^2, where r is the distance along the axis of beam. It agrees well with some experimental, however, the conventional theories based on near axis approximation, where exikonal is expanded up to only the first power in r^2, cannot account for changing radial profile of the laser beam form initial Gaussian to that of ring, particularly, if the initial intensity is sufficiently larger than the threshold, then near axis approximation fails after some distance of propagation when intensity has fallen due to defocusing. This premise has been recently applied by Liu and Tripathi [26] to investigate the ring formation in tunnel ionization of a gas by a laser beam. Subsequently, Faisal et al. [27] developed higher order paraxial theory of self-focusing in a plasma characterized by significant collisional or ponderomotive nonlinearity considering the terms up to r^4 in the expansion of eikonal and irradiance. The higher order PR theory have been concerned about researching nonlinear beam propagation and found some new abnormal self-focusing in laser-plasma interaction. Recently, Sharma et al. [28,29] and Sodha et al. [30] investigated beam self-focusing in laser-plasma interaction, due to the influence of off-axis components in higher order paraxial region, they found that the

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single highly localized beam intensity would divide into two-splitted intensity profile.

Recently, plasma with quasi-periodic density ripple has been observed [31,32] in some experiment of laser-target, due to have some novel and potential applications, the research of laser and rippled plasma interaction has been quickly developed. Recently, the plasma has found applications in the enhancement of efficiency of relativistic harmonic generation through proper phase matching, direct acceleration and interaction of ultrashort laser pulses with relativistic electron beams [33,34]. Because the plasma is a novel and special plasma by recent experimental discovery, in which beam self-focusing in a plasma with quasi-periodic density ripple has received relatively less attention. In the paper, we apply PR theory and WKB approximation to investigate off-axial contribution of beam self-focusing in plasma with periodic density ripple. At present, laser and plasma parameters as follow, laser intensity $I \approx 10^{16}$ W/cm^2, laser wavelength $\lambda \approx 1064$ nm and laser frequency $\omega_0 \approx 10^{15}$ rad/s, initial beam width $r_0 \approx 10\,\mu$m, the electron density $n_0 \approx 10^{20}$/cm^3 in the absence of laser beam, critical density $n_c \approx 10^{21}$ cm^{-3} and electron density meet $n_e < n_c$, hence, the present plasma system is an underdense plasma.

2. Analysis

A high power laser beam of frequency ω_0 and the wave vector k_0 is considered to be propagating in hot, collisionless, underdense and plasma with density ripple in the z direction. The initial intensity distribution $EE^* = \left|\vec{E}_0\right|^2 \exp(-r^2/r_0^2)$ and wave number of the beam $k_0 = \omega_0(1-\omega_p^2/\omega_0^2)^{1/2}/c$. Where $\vec{E}_0$, r, r_0 and ω_p is the laser field, the radial coordinate of the cylindrical coordinate system, initial laser beam width and the plasma wave frequency in the presence of laser beam, respectively. On account of the intensity gradient in the radial direction, ponderomotive force is generated which modifies the electron density profile and is given $N_e = n_e \exp(-3\alpha m_e EE^*/4m_i)$. Where $\alpha = e^2 m_i/6k_\beta T_0 \delta m_e^2 \omega_0^2$ is the nonlinearity parameter. n_e is electron density in the absence of the beam, k_β is Boltzman constant, δ is the ratio of the specific heats and is equal to 3, e is the electronic charge, and $m_e(m_i)$ is electron (ion) mass, and T_0 is the equilibrium plasma temperature. In the paper, the initial plasma n_e is assumed a plasma with periodic density ripple ($n_e = n_0(1 + M\cos N\xi)$). Here n_0 equilibrium plasma density in the absence of laser beam, M the modulation depth and N density modulation wave number. Assuming wave propagation in the z-direction and the electric fields in the xy-plane, the wave equation governing the beam electric vector in plasmas can be written as

$$\nabla^2 \vec{E}_0 + \frac{\omega_0^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega_0^2}\frac{N_e}{n_e}\right)\vec{E}_0 = 0 \quad (1)$$

The electric field $\vec{E}_0$ of a circularly polarized laser pulse propagation in the z-direction can be written as

$$\vec{E}_0 = \frac{1}{2}A_0(\hat{x} + i\hat{y})\exp[-i(\omega_0 t + k_0 z)] \quad (2)$$

Substituting the value of $\vec{E}_0$ in the above Eq. (2), one obtains the following equation:

$$-k_0^2 A_0 - 2ik_0 A_0 + \left(\frac{\partial^2}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial z}\right)A_0 = \frac{\omega_0^2}{c^2}\varepsilon A_0 \quad (3)$$

where A_0 is a complex function of space, ω_0, ε and c are the incident laser beam frequency, the dielectric constant and light velocity, respectively. Where the effective axial dielectric constant in higher order PR theory can be splitted into:

$$\varepsilon(r,z) = \varepsilon_0(z) - \left(\frac{r^2}{r_0^2}\right)\varepsilon_2(z) - \left(\frac{r^4}{r_0^4}\right)\varepsilon_4(z) \quad (4)$$

where ε_2 and ε_4 are the axial component of nonlinear dielectric constant. Following Akhmanov et al. [22], the complex amplitude A_0 can be expressed as:

$$A_0 = A_{00}(r,z)\exp[-ik_0 S_0(r,z)] \quad (5)$$

where $A_{00}(r, z)$ and $S_0(r, z)$ are real functions of space. Following previous work by Faisal et al. [27], the higher order paraxial theory of self-focusing in plasma characterized by significant ponderomotive or collisional nonlinearity considering the terms up to r^4 in the irradiance intensity A_{00}^2, and A_{00}^2 are expressed as follows:

$$A_{00}^2 = \left(1 + \frac{a_{02}r^2}{r_0^2 f^2} + \frac{a_{04}r^4}{r_0^4 f^2}\right)\frac{E_{00}^2}{f^2}\exp\left(-\frac{r^2}{r_0^2 f^2}\right) \quad (6)$$

The parameters a_{02} and a_{04} are indicative of off-axial component of the beam from the Gaussian nature, and E_{00}^2 is the initial laser intensity. Eq. (6) gives the intensity profile of the laser beam in the plasma along with the radial direction when ponderomotive nonlinearity is operative. In Eq. (5), $S_0(r, z)$ is eikonal function and is expressed as

$$S_0(r,z) = \frac{r^2}{2r_0^2 f}\frac{df}{dz} + \frac{S_{04}r^4}{r_0^4} \quad (7)$$

S_{04} corresponds to its departure from the spherical nature. Substituting the value of A_0 in the Eq. (3), and separating the real and imaginary parts we get the following equations:

$$2\frac{\partial S_0}{\partial z} + \left(\frac{\partial S_0}{\partial r}\right)^2 = \frac{\omega^2\varepsilon}{k_0^2 c^2} + \frac{1}{k_0^2 A_{00}}\left(\frac{1}{r}\frac{\partial A_{00}}{\partial r} + \frac{\partial^2 A_{00}}{\partial r^2}\right) \quad (8)$$

$$\frac{\partial A_{00}^2}{\partial z} + A_{00}^2\left(\frac{\partial^2 S_0}{\partial r^2} + \frac{\partial S_0}{\partial r}\right) + \frac{\partial A_{00}^2}{\partial z}\frac{\partial S_0}{\partial r} = 0 \quad (9)$$

Substituting Eqs. (4) and (6)–(7) into Eqs. (8) and (9) and equating the coefficients of r^4 on both sides of equation, we obtain the following equation:

$$\frac{dS_{04}}{dz} = \frac{c^2}{\omega_0^2 r_0^2}\frac{(8a_{02} + 18a_{02}^2 - 21a_{02}^3)}{8\varepsilon_0 f^6} - \frac{\varepsilon_4}{2\varepsilon_0} - S_{04}\left(\frac{4}{f}\frac{df}{dz} + \frac{1}{2\varepsilon_0}\frac{d\varepsilon_0}{dz}\right) \quad (10)$$

In Eq. (6), a_{02} and a_{04} shows the off-axial components of the eikonal contributions from r^2 and r^4 coefficients in the higher order paraxial region. Applied Sharma et al. [28,29] approaches and obtain the equation for the coefficient a_{02} and a_{04}

$$\frac{da_{02}}{dz} = -16\frac{S_{04}}{r_0^2}f^2 \quad (11)$$

$$\frac{da_{04}}{dz} = \frac{8S_{04}}{r_0^2}(1 - 3a_{02})f^2 \quad (12)$$

In order to investigate off-axial contribution of beam self-focusing in plasma with periodic density ripple, we have plotted the variation of off-axial component as a function of normalized propagation distance ξ ($\xi = cz/\omega_0 r_0^2$) for the normalized laser intensity $\beta E_{00}^2 = 4$, initial beam width $r_0 = 10\mu m$ and periodic density ripple $n_e = n_0(1 + 0.1\cos 0.1\xi)$. In Fig. 1, it is noted, both a_{02} and a_{04} presents a quasi-periodic oscillatory variation rather than a constant. Off-axial component a_{02} oscillate between positive and negative interval nearly [−0.0125, 0.0125] and presents a