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Coupled tapering/uptapering of dark soliton pair in nonlinear media

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ABSTRACT

Tapering/uptapering of coupled dark solitons of two coaxially co-propagating mutually incoherent beams in a nonlinear medium in presence of small gain or loss has been investigated in this paper using Beam Propagation Method (BPM), i.e., by direct simulations of the underlying Nonlinear Schrödinger Equation (NLSE). The results are compared with the earlier published results of tapering/uptapering of coupled bright solitons. It is found that in contrast to uptapering of bright solitonic pair, dark solitonic pair tapers in presence of losses. Moreover, in contrast to the possibility of tapering as well as uptapering of a bright solitonic pair, a dark solitonic pair never undergoes uptapering.

1. Introduction

Solitons have been a subject of high interest in many areas of fundamental and applied physics. In optics, solitons are said to be formed when an optical wave packet propagates (in a nonlinear medium) without broadening. The soliton would be a temporal/spatial soliton if the wave packet is a pulse/beam. Temporal/spatial solitons are due to a balance between dispersion/diffraction and nonlinear compression/focusing effects [1]. Both bright and dark, temporal as well as spatial solitons are possible in nonlinear media. A lot of investigations are focused on the study of spatial optical solitons (i.e. self-trapped beams) as those are considered indispensable in all-optical manipulations. The present analysis is focused on spatial dark solitons.

Spatial dark optical solitons are either a one-dimensional (1-D) dark stripes [2–7] or two-dimensional (2-D) dark holes [8–11] resulting from a phase singularity or an amplitude depression in their optical field. Spatial dark optical solitons are possible when the beam collapse is counteracted by light-induced self-defocusing. A few recent investigations have studied the behavior of dark solitons at nonlinear interfaces also [12,13]

In addition to the phenomenon of spatial soliton formation, one important phenomenon that should be crucial in all-optical manipulations is self-tapering/uptapering of spatial solitons. Self-tapering/uptapering of spatial solitons is the only means of all-optical control of width of an optical beam without using any fabricated structure [14]. Self-tapering/uptapering of solitons has been predicted/investigated in past for one bright solitonic beam in different media like Kerr medium [14], saturable medium [15,16], in elliptic core fiber [17] and cubic-quintic medium [18].

In addition to tapering/uptapering of one beam, coupled tapering/uptapering of two bright solitonic beams coaxially co-propagating in a nonlinear medium with small gain or loss have also been investigated recently [19,20].

Dark solitons are considered to be robust compared to bright solitons and therefore, it is worth to explore coupled tapering/uptapering of two dark solitonic beams coaxially co-propagating in a nonlinear medium.

Using mathematical treatment of [21,22], the present paper investigates coupled tapering/uptapering of two 1-D dark solitonic beams by direct simulations of the underlying Nonlinear Schrödinger Equation (NLSE) considering different physical situations. The results obtained in the present investigation are compared with the results reported in [20] for tapering/uptapering of coupled bright solitons.

2. Propagation equations using Beam Propagation Method (BPM)

In BPM, the underlying propagating equations for two coupled beams co-propagating in a loss less/gain less Kerr medium are expressed as [21,22]

$$2ik_1\frac{\partial E_1}{\partial z}+\frac{\partial^2 E_1}{\partial s^2}+2\left(\frac{k_1^2 n_2}{n_{01}}\right)\times(|E_1|^2+\kappa|E_2|^2)E_1=0 \quad (1)$$

$$2ik_2\frac{\partial E_2}{\partial z}+\frac{\partial^2 E_2}{\partial s^2}+2\left(\frac{k_2^2 n_2}{n_{02}}\right)\times(|E_2|^2+\kappa|E_1|^2)E_2=0 \quad (2)$$

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The above NLSE Eqs. (1) and (2) can be normalized by using the following transformation:

$$A_i = \sigma_0 k_1 \left(\frac{|n_2|}{n_{01}}\right)^{1/2} E_i, \quad i = 1, 2 \tag{3}$$

$$\sigma = \frac{s}{\sigma_i}, \quad \xi = \left(\frac{1}{K_1 \sigma_i^2}\right) z \tag{4}$$

where σ_i is a parameter related to the beam width. Substituting Eqs. (3) and (4) in Eqs. (1) and (2), we obtain;

$$i\frac{\partial A_1}{\partial \xi} + \frac{1}{2}\frac{\partial^2 A_1}{\partial \sigma^2} + \text{sign}(n_2)(|A_1|^2 + \kappa|A_2|^2)A_1 = 0 \tag{5}$$

$$\beta i\frac{\partial A_2}{\partial \xi} + \frac{1}{2}\frac{\partial^2 A_2}{\partial \sigma^2} + \text{sign}(n_2)\alpha(\kappa|A_1|^2 + |A_2|^2)A_2 = 0 \tag{6}$$

Considering, $k_2/k_1 = \beta$ and $\left(\frac{k_2}{k_1}\right)^2 \frac{n_{01}}{n_{02}} = \beta\left(\frac{\lambda_1}{\lambda_2}\right) = \alpha$

The sign of n_2 defines non-linear characteristics of the medium. It is negative ($n_2 < 0$) for defocusing medium, while positive ($n_2 > 0$) for focusing medium. As dark solitons are possible only in defocusing medium, we consider $n_2 < 0$ in the present paper.

For $n_2 < 0$, the solitonic solutions of Eqs. (5) and (6) are tangent hyperbolic and given by:

$$A_1 = U_{01} \ \tanh\left(\frac{s}{\sigma_1}\right) \exp(i\beta_1 \xi) \tag{7}$$

$$A_2 = U_{02} \ \tanh\left(\frac{s}{\sigma_2}\right) \exp(i\beta_2 \xi) \tag{8}$$

where

$$U_{01}^2 = \text{sign}(n_2)\frac{\kappa - \alpha}{\alpha(1-\kappa^2)} \tag{9}$$

$$U_{02}^2 = \text{sign}(n_2)\frac{\kappa\alpha - 1}{\alpha(1-\kappa^2)} \tag{10}$$

with $\beta_1 = -1$, $\beta_2 = -1/\beta$. Here, U_{01}, U_{02} are the axial electric fields of the two beams.

To know the evolution of the fields' envelopes of the two beams along the propagation direction, Eqs. (5) and (6) are solved using split step Fourier method [23,24].

However, if the medium is not loss less, the set of equations describing two coupled beams co-propagating in a medium with finite loss or gain is:

$$i\frac{\partial A_1}{\partial \xi} + \frac{1}{2}\frac{\partial^2 A_1}{\partial \sigma^2} + \gamma_1 A_1 + \text{sign}(n_2)(|A_1|^2 + \kappa|A_2|^2)A_1 = 0 \tag{11}$$

$$\beta i\frac{\partial A_2}{\partial \xi} + \frac{1}{2}\frac{\partial^2 A_2}{\partial \sigma^2} + \gamma_2 A_2 + \text{sign}(n_2)\alpha(\kappa|A_1|^2 + |A_2|^2)A_2 = 0 \tag{12}$$

Here γ_1, γ_2 signify gain or loss of the two beams depending on their positive or negative signs respectively.

3. Results and discussion

Creation of soliton pair (coupled self-trapped beams) is essential to investigate coupled self-tapering/uptapering phenomenon. Therefore, formation of dark soliton pair has been investigated in the first part of the paper using Beam Propagation Method (BPM) [23,24] by solving Eqs. (5) and (6). Being solitonic solutions, tangent hyperbolic profile of the beams is considered for present investigations.

The parameters needed for coupled solitonic propagation can be determined from Eqs. (9) and (10) which are, $\sigma_1 = \sigma_2 = 1$, $\kappa = 2$, $\alpha = 1$, $U_{01} = U_{02} = \sqrt{0.3333}$. With these parameters, Eqs. (5) and (6) give solitonic solutions of equal peak intensities $|U_{0i}|^2$ as shown

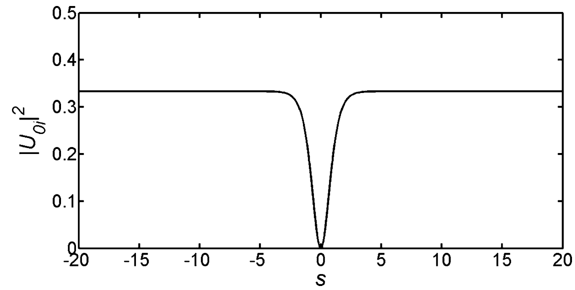

Fig. 1. Solitonic solutions of equal peak intensities $|U_{0i}|^2$ are plotted using BPM. Profile is tangent hyperbolic.

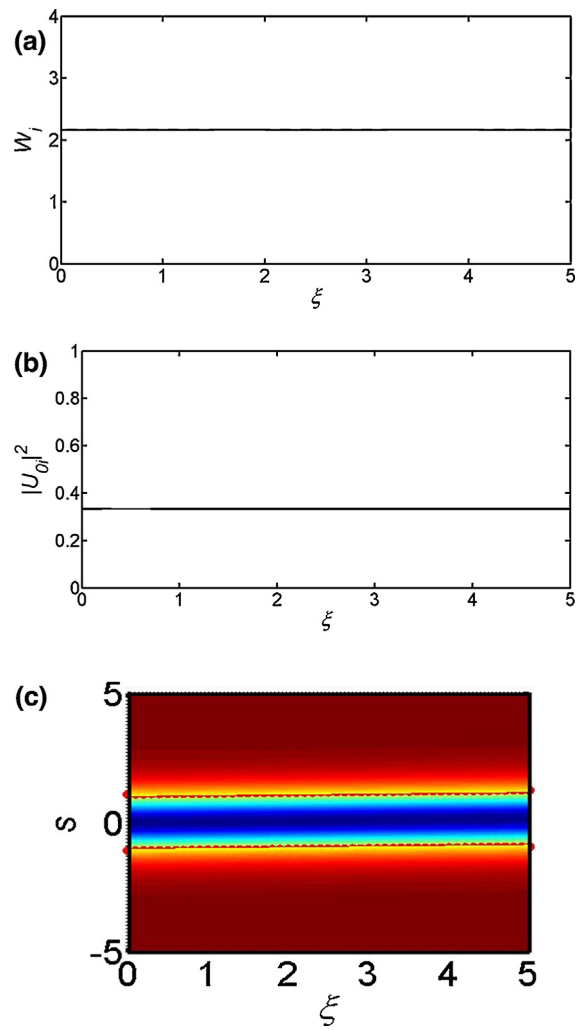

Fig. 2. (a) Dark soliton pair is created by considering $\kappa = 2$, $\sigma_1 = \sigma_2 = 1$, $\alpha = 1$, $U_{01} = U_{02} = \sqrt{0.3333}$. With these parameters, Eqs. (5) and (6) give evolution of beam widths ($W_i = 2 \times \sigma_i$) with the distance of propagation as shown in the figure. Beam widths are overlapping. Constant beam widths confirm the formation of dark solitonic pair. (b) Variation of beams' intensities $|U_{0i}|^2$ of dark soliton pair with distance of propagation is shown in the figure. (c) Mesh plot using BPM is shown in the figure. The dotted lines show $1/e$ width of the beam. The other solitonic beam propagates exactly in the same manner.

in Fig. 1. Evolution of beam widths ($W_i = 2 \times \sigma_i$) with the distance of propagation using Eqs. (5) and (6) is shown in Fig. 2(a). In the figure, overlapping W_1 and W_2 are constant with the distance of propagation, which confirm the formation of a dark solitonic pair. Fig. 2(b) shows the variation of beam intensity with the distance of propagation. Mesh plot of first beam is shown in Fig. 2(c). The