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Propagation of cosh Gaussian laser beam in plasma with density ripple in relativistic – ponderomotive regime

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ABSTRACT

Self-focusing of cosh Gaussian laser beam in plasma with periodic density ripple has been investigated. The pondermotive force on electron and the relativistic oscillation of the electron mass causes periodic self-focusing/defocusing of the cosh Gaussian laser beam. The beam converges in the region of high plasma density due to dominance of self-focusing effect over diffraction effect and diverges in the low density region. Non-linear partial differential equation governing the evolution of complex envelope in slowly varying approximation is solved using paraxial ray approximation. The variation of beam-width parameter is studied with distance of propagation for different values of ripple wave number *d* and decentred parameter *b*. In order to get strong self-focusing, wavelength and intensity parameters of cosh Gaussian laser beam are optimized.

1. Introduction

The self-focusing of ultra-intense short pulses has been a captivating topic for many researchers both from the standpoint of theoretical and experimental work due to their wide applications in X-ray lasers, laser-driven particle accelerators, plasma based accelerators and high harmonic generation [1–4]. An extensive literature is available on laser pulse propagation through plasma with various density profiles. Tripathi et al. [5] studied plasma channel charging and ion coulomb explosion in fast rising flat top short laser pulse produced plasmas. Habibi et al. [6] investigated the interaction of intense laser pulse with cold quantum plasma using ramp density profile. Also from the studies of various papers on self-focusing it was observed that high power laser propagating through plasma can acquire a minimum spot size due to relativistic self-focusing. Beyond the focus, the spot size of the laser increases due to weakening of non-linear refraction and hence showing oscillatory self focusing/defocusing. So Gupta et al. [7] introduced a slowly increasing plasma density ramp to enhance the self-focusing effect. Later on Sadighi-Bonabi et al. [8] showed that with proper plasma density ramp profile and due to relativistic effect the spot size oscillations of the laser beam increase which cause the laser beam to become more focused. However another way to increase self-focusing effect is with density ripple in plasma. Different lasers are used to create suitable density ripples in plasma. Lin et al., and Kuo et al. [9,10] reported resonant dependence of harmonic intensity on plasma density. Kaur et al. [11] studied the self-focusing of Gaussian beam using plasma density ripple. Parashar et al. [12] studied second-harmonic generation of laser radiation in plasma with density ripple. Liu et al. [13] used intense short laser pulse through plasma with density ripple for resonant third harmonic generation.

The purpose of the present study is to analyze the self-focusing of cosh-Gaussian laser beam in plasma with a sinusoidal density ripple $n_o = n_0^o(1 + \alpha_2 \cos qz)$ and optimize the laser and plasma parameters for stronger self-focusing. The cosh-Gaussian laser beam is taken because of its important technological issues, as this beam is highly powerful than Gaussian laser beam [14]. The non-linearity we have taken here is due to both relativistic and pondermotive effects. When high power laser beam is involved, it can cause an electron oscillatory velocity comparable to the velocity of light, which modifies the effective dielectric constant of the plasma. This mechanism is responsible for relativistic self-focusing of laser beam. If frequency of the beam exceeds the natural frequency of the electron oscillations in plasma, the pondermotive forces come into play. The pondermotive force of the focused laser beam pushes the electrons out of the beam field from high intensity region to low intensity region and thus reduces the local electron density which

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results in focusing of laser beam. The structure of the paper is as follows: In Section 2 we have discussed the intensity profile of cosh Gaussian laser beam in the paraxial ray approximation. The wave equation governing the spot size of the laser is derived in Section 3 where the evolution of spot size is discussed, numerical results are discussed in Section 4 and conclusion is drawn in Section 5.

2. Formalism

Consider the plasma of sinusoidal electron density n_o

$$n_o = n_o^o(1 + \alpha_2 \cos qz) \tag{1}$$

where n_o^o is the maximum electron density, α_2 is the depth of modulation and q is the ripple vector.

A cosh Gaussian laser beam propagates through the plasma along $\hat{z}$ with angular frequency ω_o. The electric vector is given as

$$\mathbf{E} = \hat{x}A(r, z)\exp(-\iota(\omega_o t - k_o z)) \tag{2}$$

where $k_o = ((\omega_o\sqrt{\epsilon_o})/(c))$ is the propagation constant of the wave, ϵ_o is the linear part of the dielectric constant of the medium and c is the speed of light in vacuum.

$$A(r, 0) = A_{oo}\exp\left(\frac{-r^2}{r_o^2}\right)\cosh(\Omega_o r) \tag{3}$$

is the field distribution of the cosh Gaussian beam at the plane of $z = 0$ as described by [15–17], where A_{oo} is the amplitude at the central position of r i.e. at $r = 0$, r_0 is the beam-width and Ω_o is the parameter associated with the hyperbolic cosine function.

Also for $z > 0$, the field distribution is

$$A(r, z) = \frac{A_{oo}}{2f}\exp\left(\frac{b^2}{4}\right)\left[\exp\left\{-\left(\frac{r}{fr_o} + \frac{b}{2}\right)\right\}^2 + \exp\left\{-\left(\frac{r}{fr_o} - \frac{b}{2}\right)\right\}^2\right] \tag{4}$$

where $b = \Omega_o r_o$ is the decentred parameter also called the normalized modal parameter and f is the dimensionless beam-width parameter of the beam in medium.

In the axial region, this profile has the form

$$A_o = \frac{A_{oo}}{f}\left[1 - \frac{r^2}{f^2 r_o^2} + \frac{r^4}{2f^4 r_o^4}\right] \times \left[1 + \frac{b^2 r^2}{2f^2 r_o^2} + \frac{b^4 r^4}{24 f^4 r_o^4}\right] \tag{5}$$

The pondermotive force on the electrons modifies the electron density and is given by

$$\vec{F_p} = -(mc^2)\vec{\nabla}(\gamma - 1) \tag{6}$$

where $\gamma = (1 + ((a^2)/2))^{1/2}$ is the relativistic factor arising from the intensity dependence of electron mass, $a = (e|A|)/(m\omega_o c)$ is the normalized laser amplitude at $z > 0$ and

$$a_o = \frac{eA_{oo}}{m\omega_o c} = \left[\frac{I(\mathrm{W\,cm^{-2}})\lambda^2(\mu\mathrm{m}^2)}{(1.37 \times 10^{18})}\right]^{1/2}$$

is the laser intensity parameter. Also $-e$, m and n_o are the electron charge, rest mass and modified density respectively. Following Tripathi et al. [5], the modified electron density can be written as

$$n_e = n_o^o[1 + \alpha_2\cos(qz)] + \frac{mc^2}{4\pi e^2}\nabla^2(\gamma - 1) \tag{7}$$

The dielectric permittivity of the plasma is given as

$$\epsilon = 1 - \frac{\omega_p^2}{\omega_o^2}\frac{(n_e/n_o^0)}{\gamma} \tag{8}$$

where $\omega_p^2 = ((4\pi n_o^o e^2)/m)$. General form of dielectric permittivity as a series of r^2 in the paraxial approximation $r^2 \ll r_o^2 f^2$ is

$$\epsilon = \epsilon_o - \left(\frac{\phi r^2}{r_o^2}\right) \tag{9}$$

Expanding n_e and γ around $r = 0$, and using Eqs. (7) and (8), we get

$$\epsilon_o = 1 - \frac{\omega_p^2}{\omega_o^2\gamma_o}\left((1 + \alpha_2\cos qz) + \frac{a_o^2 c^2(b^2 - 4)}{2f^4 r_0^2\gamma_o\omega_p^2}\right) \tag{10}$$

Here γ_o is γ at $r = 0$ and

$$\phi = \frac{\omega_p^2 a_o^2}{2\omega_o^2\gamma_o^3 f^4}(1 + \alpha_2\cos qz) + \frac{a_o^2 c^2[(4 - b^2) + ((b^2 - 3)a_o^2/f^2)/(\gamma_o^2))]}{\omega_o^2\gamma_o^2 f^6 r_o^2} \tag{11}$$

3. Self-focusing

The non-linear wave equation governing the evolution of electric field in the plasma is

$$\nabla^2\mathbf{E}(r, z) + \frac{\omega_o^2}{c^2}\epsilon\mathbf{E}(r, z) = 0, \tag{12}$$

Putting **E** from Eq. (2) in Eq. (12) and using WKB approximation, the wave equation becomes

$$2\iota k_o\frac{\partial A}{\partial z} + \nabla_\perp^2 A - \frac{r^2}{r_o^2}\frac{\omega_o^2}{c^2}\phi A = 0 \tag{13}$$

We now introduce an eikonal, $A = A_o(r, z)\exp[\iota k_o S(r, z)]$, where $A_o(r, z)$ and $S(r, z)$ are real functions of space variables. On substituting the expression for A in Eq. (13) and separating real and imaginary parts of the resulting equation, one obtains

$$2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{k_o^2 A_o}\nabla_\perp^2 A_o - \frac{r^2}{r_o^2}\frac{\phi}{\epsilon_o}, \tag{14}$$

$$\frac{\partial A_o^2}{\partial z} + \frac{\partial A_o^2}{\partial r}\frac{\partial S}{\partial r} + A_o^2\left(\frac{1}{r}\frac{\partial S}{\partial r} + \frac{\partial^2 S}{\partial r^2}\right) = 0. \tag{15}$$

Here the term $(\partial^2 A/\partial z^2)$ has been neglected as we have assumed the field to be slowly converging or diverging. Following Akhmanov et al. [18], we expand eikonal S, in the paraxial ray approximation as

$$S(r, z) = S_o(z) + S_2(z)\frac{r^2}{2} \tag{16}$$

and

$$S_2(z) = \frac{1}{f(z)}\frac{df(z)}{dz} \tag{17}$$

On substituting A_o and $S(r, z)$ from Eqs. (5) and (16) into Eq. (14) and on equating the coefficient of r^2 on both sides, the equation governing beam-width parameter is obtained as,

$$\frac{d^2 f}{d\zeta^2} = \frac{(12 - 12b^2 - b^4)}{3f^3} - \frac{\phi f\omega_o^2 r_o^2}{c^2}. \tag{18}$$

where

$$\phi = \frac{\omega_p^2 a_o^2}{2\omega_o^2\gamma_o^3 f^4}(1 + \alpha_2\cos d\zeta) + \frac{a_o^2 c^2[(4 - b^2) + (((b^2 - 3)a_o^2/f^2)/(\gamma_o^2))]}{\omega_o^2\gamma_o^2 f^6 r_o^2}.$$