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The physical optics integral on the scatterer's unlit surface

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ABSTRACT

The scattering integrals of the modified theory of physical optics are redefined according to the illuminated and unlit surfaces of the scattering object. With this aim the canonical problem of wedge diffraction is taken into account. It is shown that the new scattering integral contain two geometrical optics and diffracted fields. One of the geometrical optics waves is the reflected field component that propagates in the real space. The other one transmits to an imaginary space through the scattering surface and does not have any influence in the real space. The diffracted waves exist in the real space and satisfy the related boundary condition on the scattering surfaces. The resultant field expressions are compared with the exact series solution of the problem numerically.

1. Introduction

The physical optics (PO), which was first introduced by Macdonald in 1913 [1], is based on the integration of an induced surface current density over the illuminated part of the scattering object. Zero current density is proposed on the shadowed surface. The current is evaluated from the incident and reflected geometrical optics (GO) waves in terms of the magnetic field intensity for the perfectly conducting surfaces. As a result of this approach, the diffracted fields, which are found by the edge point technique, are not correct [2]. The second problem of PO is the exclusion of the unlit surface of the scatterer, since the GO waves that determine the surface current are zero on the shadow region of the scattering problem. For this reason, the method of PO handles the diffraction problem of waves by a wedge as a half-plane problem [3]. We had fixed first defect of PO by constructing the surface currents according to three axioms and showed that it was possible to obtain the exact solution of the scattering problem of waves by a perfectly conducting half-plane directly with PO [4,5]. Later we justified physically and mathematically the axioms that led to the rigorous solution [6,7]. The modified theory of physical optics (MTPO) was also applied to the wedge diffraction problem and it was shown that the exact diffracted fields could be obtained by the asymptotic evaluation of the scattering integrals [8]. The scattering integral of MTPO includes one GO and diffracted field. The diffracted wave is directly related with the GO field and compensates its discontinuity at the transition region where the amplitude of the GO wave suddenly falls to zero. This property of the MTPO integral represents a mathematical basis for the ideas of Young [9,10].

As mentioned above, the second problem of PO is the definition of the induced surface current only on the illuminated surface of the scatterer. The reason of this defect lays on the fact that the PO currents are evaluated from the GO waves. Thus the effect of the unlit surface on the scattering process cannot be included in the method of PO. We separated the scattering integrals of MTPO into two parts in order to examine the influence of the scatterer's shadowed surface [11,12]. Four integral were obtained after the decomposition. Two of the integrals were expressing the reflected and transmitted waves, which were propagating in the real space of $\phi \in [0,2\pi]$, from the surface of the obstacle. The remaining ones were also reflected and transmitted waves but they were propagating in an imaginary space. However the diffracted waves of the last two integrals were also affecting the wave propagation in the real space. The diffracted fields of the wedge problem were the same with the ones that were introduced by Kouyoumjian and Pathak [13]. But a PO integral on the unlit surface of the obstacle could not be defined in the mentioned studies.

The aim of this paper is to introduce a new MTPO integral, which represents the scattering process from the shadowed portion of the scatterer. First of all we will review the general construction of an MTPO scattering integral and the decomposition process, given in [11]. We will take into account the wedge diffraction problem in order to construct an integral over the unlit surface. The general philosophy of the construction process will be outlined. The wedge diffraction problem for the case of two faces illuminated will also be studied with the new approach and the obtained diffraction field expressions will be compared with the exact series solution numerically.

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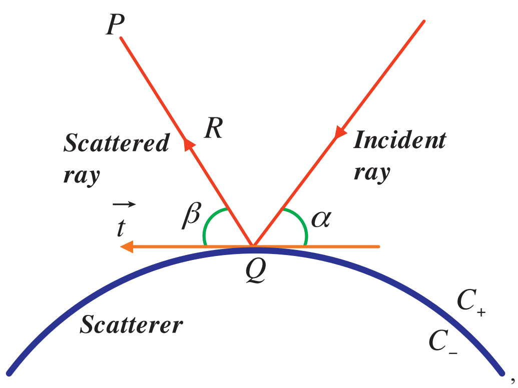

Fig. 1. The scattered ray that represents the Green's function for an arbitrary two dimensional scattering geometry.

A time factor of $e^{j\omega t}$ is taken into account and suppressed throughout the paper. j is $(-1)^{1/2}$. ω expresses the angular frequency and t is time. The Cartesian and polar coordinates are represented by (x,y) and (ρ,ϕ) for two dimensional problems.

2. Review of the MTPO surface integrals

In this section, we will review the main features of the MTPO scattering integrals and their special decomposition, introduced in [11]. The scattered waves by a perfectly conducting surface, given in Fig. 1 can be expressed by the integrals of

$$u_{ts}(P)=\frac{ke^{j(\pi/4)}}{\sqrt{2\pi}}\int_{C_+}u_i(Q)\sin\frac{\beta-\alpha}{2}\frac{e^{-jkR}}{\sqrt{kR}}dl' \quad (1)$$

and

$$u_{rs}(P)=\mp\frac{ke^{j(\pi/4)}}{\sqrt{2\pi}}\int_{C_+}u_i(Q)\sin\frac{\beta+\alpha}{2}\frac{e^{-jkR}}{\sqrt{kR}}dl' \quad (2)$$

for a two dimensional problem [6].

P and Q are the observation and integration (scattering) points. k is the wavenumber. α and β are the angles of incidence and scattering at Q respectively. u_i is the z component of the incident electric or magnetic field intensity. Note that the z axis is directed out of the paper. The + and − signs, in Eq. (2), are valid for the z polarized incident electric and magnetic field intensities respectively. C_+ is the integration contour along the illuminated side of the scatterer and dl' is the length element on C_+. C_- shows the shadowed part of the scattering surface. The subscripts ts and rs represent transmitted scattered and reflected scattered respectively. R is the distance between the points of integration and observation. $\vec{t}$, in Fig. 1, is the unit tangent vector of the surface at Q. The Green's function of the integrals, in Eqs. (1) and (2), is given by

$$G(P,Q)=\sin\frac{\beta\mp\alpha}{2}\frac{e^{jkR}}{\sqrt{kR}}, \quad (3)$$

which satisfies the Helmholtz equation of

$$R\frac{\partial}{\partial R}\left(R\frac{\partial G}{\partial R}\right)+\frac{\partial^2 G}{\partial\beta^2}+k^2R^2G=0 \quad (4)$$

at point Q. Thus the scattering integrals, in Eqs. (1) and (2), are also the solution of the Helmholtz equation. The stationary phase points of these scattering integrals occur at $\beta_s=\pm\alpha$ [5,6]. β_s is the value of β at the stationary phase point. The integral, in Eq. (1), is equal to zero at $\beta_s=\alpha$. It is nonzero for $-\alpha$, which shows a ray that passes through the scattering surface. Thus the first scattering integral represents the waves that transmit through the scatterer's surface as if it is transparent. The amplitude of the transmitted GO wave, which is evaluated by the stationary phase method, is equal to the incident wave multiplied by −1. The total transmitted field becomes zero in the shadow of the scatterer when summed with the incident field. Note that the total scattered wave has the expression of

$$u_s(P)=u_i(P)+u_{ts}(P)+u_{rs}(P). \quad (5)$$

The scattering integral, in Eq. (2), is nonzero for $\beta_s=\alpha$, which represents a ray that reflects from the scattering point Q. The classical PO integral can also be divided into two parts that represents the transmitted and reflected scattered waves, but their asymptotic evaluation leads to incorrect diffraction field expressions [14].

Now we will attempt to construct new expressions for the scattering integrals that are defined on the lit (C_+) and shadowed (C_-) portions of the scatterer separately. Note that Eqs. (1) and (2) are only written for the illuminated part of the scatterer although they include the field expressions in the shadow region. With this aim, we take into consideration the relation of

$$\sin\frac{\beta\mp\alpha}{2}=-\frac{1}{4}(\cos\beta-\cos\alpha)\left(\cot\frac{\beta\pm\alpha}{4}+\tan\frac{\beta\pm\alpha}{4}\right) \quad (6)$$

for the term $\cos\alpha-\cos\beta$ is the first derivative of the phase function of the scattering integrals, in Eqs. (1) and (2) [11]. We obtain the integrals of

$$u_{ts1}(P)=-\frac{ke^{j(\pi/4)}}{4\sqrt{2\pi}}\int_{C_+}u_i(Q)(\cos\beta-\cos\alpha)\cot\frac{\beta+\alpha}{4}\frac{e^{-jkR}}{\sqrt{kR}}dl', \quad (7)$$

$$u_{ts2}(P)=-\frac{ke^{j(\pi/4)}}{4\sqrt{2\pi}}\int_{C_+}u_i(Q)(\cos\beta-\cos\alpha)\tan\frac{\beta+\alpha}{4}\frac{e^{-jkR}}{\sqrt{kR}}dl', \quad (8)$$

$$u_{rs1}(P)=\pm\frac{ke^{j(\pi/4)}}{4\sqrt{2\pi}}\int_{C_+}u_i(Q)(\cos\beta-\cos\alpha)\cot\frac{\beta-\alpha}{4}\frac{e^{-jkR}}{\sqrt{kR}}dl' \quad (9)$$

and

$$u_{rs2}(P)=\pm\frac{ke^{j(\pi/4)}}{4\sqrt{2\pi}}\int_{C_+}u_i(Q)(\cos\beta-\cos\alpha)\tan\frac{\beta-\alpha}{4}\frac{e^{-jkR}}{\sqrt{kR}}dl' \quad (10)$$

when Eq. (6) is used in Eqs. (1) and (2). The nonzero value of the first integral's amplitude function is at $\beta_s=\alpha$. Eq. (7) is equal to zero for all other stationary phase values of β because of the term $\cos\beta-\cos\alpha$. Thus Eq. (7) represents the reflected scattered wave from the lit portion of the scatterer (C_+). The integral, in Eq. (10), is nonzero only for $\beta_s=2\pi-\alpha$ and expresses the transmitted wave to an imaginary space through the lit surface of C_+. The real space is defined by the region $\beta\in[-\pi,\pi]$ with respect to β. For this reason, the stationary phase value of $2\pi-\alpha$ is out of the real space. In this paper, we will not deal with the integrals, in Eqs. (8) and (9). We can write a new scattering integral of

$$u_{s1}(P)=-\frac{ke^{j(\pi/4)}}{4\sqrt{2\pi}}\int_{C_+}u_i(Q)\left(\tan\frac{\beta+\alpha}{4}\mp\cot\frac{\beta-\alpha}{4}\right)\times(\cos\beta-\cos\alpha)\frac{e^{-jkR}}{\sqrt{kR}}dl' \quad (11)$$

over the illuminated portion of the scatterer. Eq. (11) includes all the scattered fields that have interaction with the illuminated side. There are two possible field interactions with a surface. The wave reflects from the surface and propagates in the real space or it transmits through the scatterer and enters an imaginary space. The second wave is a mathematical result of PO. However Eq. (11) does not contain the entire story. As mentioned above, we have a second wave that transmits through the scatterer and penetrates the shadow region of the problem. This component was named as the shadow radiation by Ufimtsev, but he could not obtain a unique expression for the transmitted field as given in Eq. (1) [15,16]. Thus