Optik 125 (2014) 5134–5140

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Entropy squeezing of multi-photon field interacting with a single Cooper-pair box[☆]

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ARTICLE INFO

Article history:
Received 14 September 2013
Accepted 26 April 2014

Keywords:
Entanglement
Entropy squeezing
Mutual entropy
Single cooper-pair box
Multi-photon field

ABSTRACT

We investigate the entropy squeezing and entanglement of the Cooper-pair box interacting with multi-photon cavity field. The field is prepared initially in the coherent state, while the Cooper-pair box is assumed to start from a mixed state. We find that the number of photons and the detuning parameters play an important role in the entropy squeezing and entanglement. We observe that the entropy squeezing can be used as a good indicator of the entanglement. This study opens promising perspectives for creating remote quantum information processing networks.

1. Introduction

Quantum entanglement is a fascinating subject which reveals the fundamental difference between quantum and classical phenomenons. Recently, entanglement, as a physical resource, is used in information processing, communication and quantum computing [1,2], such as information entropy [3], the behavior of charge oscillations [4,5], the dynamics of coded information [6], the dynamics of the skew information [7] and Quantum teleportation [8]. On the other hand, several efforts have been made to quantify the entanglement between particles and fields. To do so, one has to know whether the initial states are pure states or mixed states. That is, if the entangled states are pure states, it is sufficient to use Von Neumann entropy which has a unique measurement [9]. Otherwise, a suitable measure of entanglement will be used for the mixed state because it does not have a unique measure [10]. The dynamics of entanglement for the Jaynes–Cummings model with mixed state is discussed by Jaynes and Cummings [11,12].

Quantum entanglement have been realized in various systems including that based on photons [13], QED [14], trapped atoms [15], and solid state systems (superconducting materials [16], quantum dot [17] and silicon based quantum systems [18]), and have even been known to exist naturally in systems such as light harvesting photosynthesis [19]. Superconducting material has attracted much attention in the last century due to its importance in many applications like magnetic sensors [20,21], Josephson-junctions arrays [22] and electronics based superconducting devices [23]. In 1999 Nakamura et. al fabricated the first quantum circuit based on the superconducting material to generate quantum bit or qubit. In their study, the basic component of the quantum computer is a single Cooper-pair box [24]. They observed the quantum state of a single two-level system, i.e., a qubit, in a solid-state electronic device. Then, a suitable implementation of the first solid-state quantum processor was developed by several groups of researcher [16,25,26]. The coherent time evolution between two quantum states is observed in an experiment consists of three Josephson junctions in a loop [27]. The current progress in this field is the increasing of lifetime of the entanglement [28] and generating sufficient qubits to be stored and retrieved from the quantum memory [29].

From the quantum information theory point of view, the entropy squeezing is investigated for a single two-level atom interacting with a single mode or two modes [30,31] and the entanglement of the solid-state circuit QED [32,33]. It is noted that to study the

☆ This is only an example.
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http://dx.doi.org/10.1016/j.ijleo.2014.05.010
0030-4026/

information of the squeezing entropy, we define the quadrature variances for the atomic operator using Heisenberg uncertainty relation (HUR) which is formulated in terms of the variances of the system observable. In this regard, the degree of entanglement for mixed state is studied by some entropic measures such as quantum relative entropy [34] and entanglement of formation [35]. Also, several methods for quantifying the quantum entanglement of the multi-partite systems were presented [36]. Our aim in the present paper is to investigate the entropy squeezing and entanglement via the quantum mutual entropy as a measure of multi-photon field interacting with a single Cooper-pair box where the field and the box are prepared initially in the coherent mixed state.

This paper is prepared as follows: In Section 2, we introduce the basic model and derive the time evolution of the density matrix. In Section 3, we investigate the entropy squeezing and calculate the quadrature variances for the atomic operator of our model. We study the degree of entanglement due to the mutual entropy in Section 4 and our result is summarized in the last section.

2. The model

We consider a superconducting box consists of a single superconducting island connected by Josephson junction with energy E_J and capacitance C_J, coupled capacitively to a gate voltage V_g (gate capacitance C_g) [24,37] and placed inside a single-mode microwave field through multi-photon transition. Suppose that the gate capacitance C_g is screened from the quantized radiation field and then the system can be described by a two-level system with Hamiltonian [6,5,38–40]

$$H = \frac{(Q - C_g V_g - C_J V)^2}{2(C_g + C_J)} - E_J \cos\phi + \hbar\omega(\hat{a}^\dagger\hat{a} + t\frac{1}{2}), \tag{1}$$

where $Q = 2Ne$ is the Cooper pair charge on the island in which N is the number of Cooper-pairs, ϕ is the phase difference across the junction, ω is the field frequency, $\hat{a}^\dagger$ and $\hat{a}$ are the creation and annihilation operators of the microwave, respectively. The voltage difference, V, produced by the microwave across the junction may be written as [5,38,40,41]

$$V = i(\frac{\hbar\omega}{2C_F})^{1/2}(\hat{a} - \hat{a}^\dagger), \tag{2}$$

where C_F is the capacitance parameter. The charging energy with scale $E_c = e^2/2(C_g + C_J)$ which dominates over the Josephson coupling energy E_J is concentrated on the value $V_g = (e)/(C_g)$ and weak quantized radiation field, so that only the two low-energy charge states $N = 0$ and $N = 1$ are relevant. In this case, the new Hamiltonian can be written in the new constructed basis of the charge states, $|\uparrow\rangle$ and $|\downarrow\rangle$, which have simple form in a two-level system [42],

$$H = E_c[1 + \frac{C_J^2}{e^2}V^2] - \frac{1}{2}E_J S_x - 2E_c\,, \tag{3}$$

where S_x and S_z are Pauli operators in the pesudo-spin basis $|\uparrow\rangle, |\downarrow\rangle$ to which $S_x = |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|$ and $S_z = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|$. If the quantized radiation field is neglected, i.e., $V = 0$, the charge states are not the eigenstate of the Hamiltonian. Therefore, we can rewrite H in a two-charge state subspace and denote the corresponding states $|1\rangle$ and $|0\rangle$ as

$$|1\rangle = \frac{\sqrt{2}}{2}(|\uparrow\rangle - |\downarrow\rangle), \quad |0\rangle = \frac{\sqrt{2}}{2}(|\uparrow\rangle + |\downarrow\rangle). \tag{4}$$

In this case the Hamiltonian in a basis of the charge state $|\downarrow\rangle$ and $|\uparrow\rangle$ reduces to a two-state form in a spin $-1/2$ language [43]. In a weak quantized radiation field one may neglect the term containing V^2 in Eq. (3) and in the case where $E_J \sim \hbar\omega \ll E_c$, the Hamiltonian takes the form [38]

$$H_c = \frac{1}{2}E_J S_z\,, \tag{5}$$

$$H_f = \hbar\omega\hat{a}^\dagger\hat{a}\,, \tag{6}$$

$$H_I = \frac{1}{2}\Delta S_z - i\hbar g(\hat{a}^k S_+ - \hat{a}^{\dagger k} S_-)\,, \tag{7}$$

where

$$g = \frac{eC_J}{2(C_g + C_J)}(\frac{\omega}{2\hbar C_F})^{1/2}, \tag{8}$$

and $\Delta = E_J - k\omega$ is the detuning between the Josephson energy and multi-photon field frequency. Here, S_+ and S_- are the rasing and lowering operators, respectively, and $[S_+, S_-] = S_z$.

The evolution operator $U_t = \exp(-i\hat{H}t)$ can be written as [44]

$$U_t = \begin{pmatrix} \mathcal{U}_{11} & \mathcal{U}_{12} \\ \mathcal{U}_{21} & \mathcal{U}_{22} \end{pmatrix}. \tag{9}$$

We assume the Cooper-pair box at $t = 0$ is prepared in a mixed state,

$$\rho_b(0) = \beta|0\rangle\langle 0| + (1 - \beta)|1\rangle\langle 1|\,. \tag{10}$$

The field is initially in a coherent state $\rho_f(0) = |\alpha\rangle\langle\alpha|$ where

$$|\alpha\rangle = \sum_{n=0}^{\infty} C_n| \quad n\rangle, \tag{11}$$

$$C_n = \exp(-\frac{\overline{n}}{2})\frac{\overline{n}^{n/2}}{\sqrt{n!}}\exp(in\beta)\,, \tag{12}$$

where $\overline{n}$ and β are the initial average photon and the phase angle of the field, respectively.

The density operator of the system at $t > 0$ is given by $\hat{\rho}(t) = U_t \otimes \hat{\rho}(0) \otimes U_t^\dagger$, where $\rho(0) = \rho_b(0) \otimes \rho_f(0)$. By using Eqs. (7)–(9), the elements of the density operator are given by

$$\rho_{11}(t) = (1-\beta)\sum_{n=0}^{\infty}\frac{e^{-|\alpha|^2}|\alpha|^{2n}}{n!}[\cos^2\Omega_{n+k}t + (\frac{\Delta}{2})^2\sum_{n=0}^{\infty}\frac{\sin^2\Omega_{n+k}t}{\Omega_{n+k}^2}] + g^2\beta\sum_{n=0}^{\infty}\frac{e^{-|\alpha|^2}|\alpha|^{2n+2k}}{n!}(\frac{\sin^2\Omega_{n+k}t}{\Omega_{n+k}^2}), \tag{13}$$

$$\rho_{12}(t) = -g(1-\beta)\sum_{n=0}^{\infty}\frac{e^{-|\alpha|^2}|\alpha|^{2n+k}}{n!}[\frac{\cos\Omega_{n+2k}t\sin\Omega_{n+k}t}{\Omega_{n+k}} - i(\frac{\Delta}{2})\sum_{n=0}^{\infty}\frac{\sin\Omega_{n+2k}t\sin\Omega_{n+k}t}{\Omega_{n+k}\Omega_{n+2k}}] + g\beta\sum_{n=0}^{\infty}\frac{e^{-|\alpha|^2}|\alpha|^{2n+k}}{n!} [-\frac{\sin\Omega_{n+k}t\cos\Omega_n t}{\Omega_{n+k}} + i(\frac{\Delta}{2})\sum_{n=0}^{\infty}\frac{\sin\Omega_{n+k}t\sin\Omega_n t}{\Omega_{n+k}\Omega_n}], \tag{14}$$

$$\rho_{21}(t) = g(1-\beta)\sum_{n=0}^{\infty}\frac{e^{-|\alpha|^2}|\alpha|^{2n+k}}{n!}[\frac{\cos\Omega_{n+2k}t\sin\Omega_{n+k}t}{\Omega_{n+k}} + i(\frac{\Delta}{2})\sum_{n=0}^{\infty}\frac{\sin\Omega_{n+2k}t\sin\Omega_{n+k}t}{\Omega_{n+k}\Omega_{n+2k}}] + g\beta\sum_{n=0}^{\infty}\frac{e^{-|\alpha|^2}|\alpha|^{2n+k}}{n!} [-\frac{\sin\Omega_{n+k}t\cos\Omega_n t}{\Omega_{n+k}} - i(\frac{\Delta}{2})\sum_{n=0}^{\infty}\frac{\sin\Omega_{n+k}t\sin\Omega_n t}{\Omega_{n+k}\Omega_n}], \tag{15}$$