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Quantum teleportation of arbitrary two-qubit state via entangled cavity fields

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ABSTRACT

We propose a new protocol for quantum teleportation of an arbitrary two qubit state via continuous variables entangling channel. In our scheme two pairs of entangled light fields are employed. An outstanding characteristic of this scheme is that arbitrary state of two atoms is transmitted deterministically and directly to another pair of atoms without the help of the other atoms.

1. Introduction

Recent years quantum teleportation has been extensively investigated both theoretically and experimentally due to its important application in quantum computation [1] and quantum communication [2–10]. Quantum teleportation, originally proposed by Bennett et al. in 1993, is such a technique as information is transferred from a sender to a receiver through a quantum entanglement channel with the help of some classical information. Since then, many various proposals were proposed through various quantum entanglement channels such as entangled Bell state [2], three-particle entangled state [7], entangled photon pairs, etc. quantum teleportation has been demonstrated experimentally in optical system [6], nuclear magnetic resonance [11], and trapped ions [12]. The author of Ref. [13] proposed a teleportation scheme for an arbitrary two qubit state which is based on the 16 orthogonal generalized Bell states. The author of Ref. [14] used a genuine irreducible four-qubit entangled state to transport arbitrary two-qubit state. In these proposed protocols, it is required that the multi-qubit entangled state is taken as quantum channel and the Bell state measurement technique for atomic states is used.

In this paper, we present a new quantum teleportation scheme for an arbitrary two-qubit state. In our scheme, the entanglement cavity fields instead of the entangled atomic states are taken as quantum channel and cavity quantum electric dynamics (QED) technique is used. This protocol needs not the Bell state measurement technique for atomic states and has 100% success probability.

2. Theoretical description

Let us consider a three-level atom inside a single-mode light cavity driven by a classical field with frequency ω. The atomic states are denoted by $|g\rangle$, $|e\rangle$, and $|i\rangle$. We assume that the transition frequency between the states $|e\rangle$ and $|i\rangle$ is highly detuned from the cavity field frequency. The Hamiltonian of this system is [9] ($\hbar = 1$)

$$H = \frac{\omega_0}{2}\sigma_z + \omega_c a^+ a + g(\sigma_+ a + \sigma_- a^+) + \Omega(\sigma_+ e^{-i\omega t} + \sigma_- e^{i\omega t}) \quad (1)$$

where $\sigma^+ = |e\rangle\langle g|$, $\sigma^- = |g\rangle\langle e|$, $\sigma^z = |e\rangle\langle e| - |g\rangle\langle g|$, $|e\rangle$ and $|g\rangle$ are the excited and ground states of the atom, a^+ and a are the creation and annihilation operators for the cavity mode, ω_0 is the transition frequency of the atom, g is the atom-cavity coupling strength, and Ω is the Rabi frequency of the classical field. We assume that $\omega_0 = \omega$. Then the interaction Hamiltonian, in the interaction picture, is

$$H_I = e^{-i\delta t} g^* \sigma^+ a + e^{i\delta t} g \sigma^- a^+ + \Omega(\sigma^+ + \sigma^-) \quad (2)$$

where $\delta = \omega_c - \omega_0$ is the detuning between the atomic transition frequency and cavity field frequency, here we set $\delta = 0$. By making

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a unitary transformation $H_I' = TH_IT^+ = e^{i\pi\sigma^y/4}H_Ie^{-i\pi\sigma^y/4}$, we can obtain the transformed interaction Hamiltonian

$$H_I' = \frac{1}{2}ga^+(\sigma^z - i\sigma^y) + \frac{1}{2}g^*a(\sigma^z + i\sigma^y) + \Omega\sigma^z \tag{3}$$

Making the rotating wave approximation, which is equivalent to the transformation $e^{-i2\Omega t\sigma_z}H_I'e^{-i2\Omega t\sigma_z}$, the transformed interaction Hamiltonian H_I' becomes

$$H_I' = \frac{1}{2}(ga^+ + g^*a)\sigma^z - \frac{1}{2}(g^*a - ga^+)(\sigma^+e^{i2\Omega t} - \sigma^-e^{-i2\Omega t}) + \Omega\sigma^z \tag{4}$$

Assuming that $2\Omega \gg g$, we can neglect the fast oscillating terms. Then H_I' reduces to

$$H_I' = \frac{1}{2}(ga^+ + g^*a)\sigma^z + \Omega\sigma^z \tag{5}$$

Proceeding to making the anti-transformation, $H_I = T^+H_I'T$, we can obtain

$$H_I = \frac{1}{2}(ga^+ + g^*a)\sigma^x + \Omega\sigma^x \tag{6}$$

The evolution operator for H_I is represented as

$$U = e^{-itH_I} = e^{-i\Omega t\sigma^x}e^{-i(ga^+ + g^*a)\sigma^x/2} \tag{7}$$

In next section, we will apply Eq. (7) in quantum teleportation of an arbitrary two-qubit state.

3. Teleportation of arbitrary two-bit state

We consider the teleportation of an unknown arbitrary two-qubit state $|\psi>_{12}$ using continuous variables entangled state as quantum channel. Suppose the state of atoms 1 and 2 which is teleported by Alice to Bob is an unknown arbitrary two-qubit state

$$|\psi>_{12} = a|gg>_{12} + b|ge>_{12} + c|eg>_{12} + d|ee>_{12} \tag{8}$$

where a, b are unknown coefficients, $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$. Atoms 3 and 4 belonging to Bob are prepared in the ground state. Two pairs of entangled cavity fields (cavity 1, 2 belongs to Alice and cavity 3, 4 belongs to Bob) are used as quantum channel

$$|\varphi> = \frac{(|i\alpha>_1|i\alpha>_3 + |-i\alpha>_1|-i\alpha>_3)(|i\alpha>_2|i\alpha>_4 + |-i\alpha>_2|-i\alpha>_4)}{N} \tag{9}$$

where N is normalized constant. The initial state of the whole system composed of atoms 1, 2, 3, 4 and light cavities are given by

$$|\psi(0)> = (a|gg>_{12} + b|ge>_{12} + c|eg>_{12} + d|ee>_{12}) \otimes |\varphi> \otimes |gg>_{34} \tag{10}$$

The entire teleportation protocol is described as follows:

Step 1: Alice and Bob send atoms 1, 2 and 3, 4 into each single-mode cavity 1, 2 and 3, 4, respectively. The evolution operator U_k of the kth atoms and the kth cavity field is described by Eq. (7). We have assumed that coupling parameters Ω, g between atoms and light fields are same. By selecting the interacting time t and the parameters λ, Ω to satisfy the condition $\Omega t = m\pi$, m being a large positive even integer, we can obtain the evolution operator U_k as

$$U_k = \frac{1}{2}\{[D_k(i\alpha) + D_k^+(i\alpha)] - [D_k(i\alpha) - D_k^+(i\alpha)]\sigma_k^x\} \tag{11}$$

where $D_k(i\alpha) = e^{i\alpha(a_i + a_i^+)}$ is displacement operator of light field in the kth cavity, $\alpha = gt/2$ (here it is assumed that g is real). According to Eq. (11), we can obtain the state of the whole system as

$$\begin{aligned}
|\psi> &= U_1U_2U_3U_4|\psi(0)> \\
&= \frac{1}{16}\{(a|gg>_{12} + b|ge>_{12} + c|eg>_{12} + d|ee>_{12})\,|gg>_{34}R_1^AR_1^B \\
&- (a|gg>_{12} + b|ge>_{12} + c|eg>_{12} + d|ee>_{12})|eg>_{34}R_2^AR_1^B \\
&- (a|eg>_{12} + b|ee>_{12} + c|gg>_{12} + d|ge>_{12})|gg>_{34}R_3^AR_1^B \\
&+ (a|eg>_{12} + b|ee>_{12} + c|gg>_{12} + d|ge>_{12})|eg>_{34}R_4^AR_1^B \\
&- (a|gg>_{12} + b|ge>_{12} + c|eg>_{12} + d|ee>_{12})|ge>_{34}R_1^AR_2^B \\
&+ (a|gg>_{12} + b|ge>_{12} + c|eg>_{12} + d|ee>_{12})|ee>_{34}R_2^AR_2^B \\
&+ (a|eg>_{12} + b|ee>_{12} + c|gg>_{12} + d|ge>_{12})|ge>_{34}R_3^AR_2^B \\
&- (a|eg>_{12} + b|ee>_{12} + c|gg>_{12} + d|ge>_{12})|ee>_{34}R_4^AR_2^B \\
&- (a|ge>_{12} + b|gg>_{12} + c|ee>_{12} + d|eg>_{12})|gg>_{34}R_1^AR_3^B \\
&+ (a|ge>_{12} + b|gg>_{12} + c|ee>_{12} + d|eg>_{12})|eg>_{34}R_2^AR_3^B \\
&+ (a|ee>_{12} + b|eg>_{12} + c|ge>_{12} + d|gg>_{12})|gg>_{34}R_3^AR_3^B \\
&- (a|ee>_{12} + b|eg>_{12} + c|ge>_{12} + d|gg>_{12})|eg>_{34}R_4^AR_3^B \\
&+ (a|ge>_{12} + b|gg>_{12} + c|ee>_{12} + d|eg>_{12})|ge>_{34}R_1^AR_4^B \\
&- (a|ge>_{12} + b|gg>_{12} + c|ee>_{12} + d|eg>_{12})|ee>_{34}R_2^AR_4^B \\
&- (a|ee>_{12} + b|eg>_{12} + c|ge>_{12} + d|gg>_{12})|ge>_{34}R_3^AR_4^B \\
&+ (a|ee>_{12} + b|eg>_{12} + c|ge>_{12} + d|gg>_{12})|ee>_{34}R_4^AR_4^B
\end{aligned} \tag{12}$$

where

$$R_1^\mu = 2|0,0>_\mu + |i2\alpha, i2\alpha>_\mu + |i2\alpha, 0>_\mu + |0, i2\alpha>_\mu + |0, -i2\alpha>_\mu + |-i2\alpha, 0>_\mu + |-i2\alpha, -i2\alpha>_\mu \tag{13}$$

$$R_2^\mu = |i2\alpha, i2\alpha>_\mu - |i2\alpha, 0>_\mu + |0, i2\alpha>_\mu - |0, -i2\alpha>_\mu + |-i2\alpha, 0>_\mu - |-i2\alpha, -i2\alpha>_\mu \tag{14}$$

$$R_3^\mu = |i2\alpha, i2\alpha>_\mu + |i2\alpha, 0>_\mu - |0, i2\alpha>_\mu + |0, -i2\alpha>_\mu - |-i2\alpha, 0>_\mu - |-i2\alpha, -i2\alpha>_\mu \tag{15}$$

$$R_4^\mu = 2|0,0>_\mu + |i2\alpha, i2\alpha>_\mu - |i2\alpha, 0>_\mu - |0, i2\alpha>_\mu - |0, -i2\alpha>_\mu - |-i2\alpha, 0>_\mu + |-i2\alpha, -i2\alpha>_\mu \tag{16}$$

In the above formulae, we have used symbols $\mu = A$, B, $A = (1, 3)$, $B = (2, 4)$ and abbreviation $|\alpha, \beta>_A = |\alpha>_1|\beta>_3$, $|\alpha, \beta>_B = |\alpha>_2|\beta>_4$.

We define the even and odd coherence states for the kth cavity field as

$$|\Phi^\pm>_k = \frac{(|i2\alpha>_k \pm |-i2\alpha>_k)}{\sqrt{N_1}} \quad (k = 1, 2) \tag{17}$$

where N_1 is normalized constant. By using the even and odd coherence states, Eqs. (13)–(16) can be represented as

$$R_1^\mu = 2|0,0>_\mu + \frac{N_1}{2}(|\Phi^+, \Phi^+>_\mu + |\Phi^-, \Phi^->_\mu) + \sqrt{N_1}(|\Phi^+, 0>_\mu + |0, \Phi^+>_\mu) \tag{18}$$

$$R_2^\mu = \frac{N_1}{2}(|\Phi^+, \Phi^->_\mu + |\Phi^-, \Phi^+>_\mu) + \sqrt{N_1}(|0, \Phi^+>_\mu - |\Phi^+, 0>_\mu) \tag{19}$$

$$R_3^\mu = \frac{N_1}{2}(|\Phi^+, \Phi^->_\mu + |\Phi^-, \Phi^+>_\mu) + \sqrt{N_1}(|\Phi^+, 0>_\mu - |0, \Phi^+>_\mu) \tag{20}$$

$$R_4^\mu = 2|0,0>_\mu + \frac{N_1}{2}(|\Phi^+, \Phi^+>_\mu + |\Phi^-, \Phi^->_\mu) - \sqrt{N_1}(|\Phi^+, 0>_\mu + |0, \Phi^+>_\mu) \tag{21}$$