Optik 125 (2014) 5432–5435

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Optical solitons in the parabolic law media with high-order dispersion

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ARTICLE INFO

Article history:
Received 2 October 2013
Accepted 27 May 2014

Keywords:
Parabolic law
High-order dispersion
Optical soliton
Triangular periodic solution

ABSTRACT

The transmission equation of ultrashort optical pulse in the high-order dispersion media with the parabolic law (cubic–quintic) nonlinearity has been studied with the help of the subsidiary ordinary differential equation expansion method. As a result, the optical solitons and triangular periodic solutions are obtained, and the conditions for exact solutions to exist are also given.

1. Introduction

The nonlinear Schrödinger equations are very essential for describing the propagation of optical pulse in various nonlinear media [1–8], and seeking exact solutions of these equations are important in nonlinear science fields, especially the optical solitons, which are often applied to optical communication areas since they can hold their shapes in the process of transmissions and collisions. It is well known that the optical soliton is a very graceful kind of physical process, which is a balance of dispersion or (and) diffraction effect and nonlinear effect.

The purpose of this paper is to look for analytical solutions of the following generalized nonlinear Schrödinger equation (GNLSE) [1], which describes the propagation of ultrashort optical pulse in high-order dispersion, double-doped and loss (or gain) optical fibers:

$$i\frac{\partial u}{\partial \xi} - \frac{\beta_2}{2}\frac{\partial^2 u}{\partial \tau^2} - i\beta_3\frac{\partial^3 u}{\partial \tau^3} - \frac{\beta_4}{24}\frac{\partial^4 u}{\partial \tau^4} + \gamma_1|u|^2u + \gamma_2|u|^4u = -i\frac{\Gamma}{2}u + \tau_R u\frac{\partial(|u|^2)}{\partial \tau} - is\frac{\partial(|u|^2u)}{\partial \tau} \quad (1)$$

where $u(\xi, \tau)$ is the complex envelope of the electrical field in a comoving frame, ξ and τ are the spatial and temporal variable. In the left-hand side of Eq. (1), the second, third, and fourth term represent the group velocity dispersion (GVD), third order dispersion (TOD), and fourth order dispersion (FOD), β_2, β_3 and β_4 are GVD, TOD, and FOD coefficient, respectively. The fifth and sixth term represent the Kerr and saturation nonlinearity, γ_1 and γ_2 are the Kerr nonlinear coefficient and saturation of the nonlinear refractive index coefficient. In the right-hand side of Eq. (1), the first term represents loss (or gain) of optic fibers, Γ is the loss ($\Gamma > 0$) or gain ($\Gamma < 0$) coefficient. The second and third term represent Raman and self-steepening effect, τ_R and s are the Raman and self-steepening effect coefficient, respectively.

To date, many methods have been developed to construct explicit solutions of the nonlinear equations, for instances, the homogeneous balance method [9], the Jacobi elliptic function expansion method [10], the subsidiary ordinary differential equation (sub-ODE) expansion method [11,12], the G'/G expansion method [13,14], the self-similar method [15,16], and so on. The aim of this paper is to construct exact solutions of the Eq. (1) by using the sub-ODE expansion method. Finally, the optical solitons and triangular periodic solutions of Eq. (1) are obtained, and the conditions of these solutions existed are also given.

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http://dx.doi.org/10.1016/j.ijleo.2014.06.017
0030-4026/

2. The sub-ODE and its solutions

In this paper, we use the following first-order nonlinear ordinary differential equation (also known as the Ricatti equation) as the sub-ODE:

$$\frac{d\varphi}{d\eta} = a + b\varphi^2(\eta) \tag{2}$$

where a and b are non-zero real constants.

Eq. (2) possesses soliton solutions and triangular periodic solutions, which are listed as follows:

Case A: Suppose that $ab > 0$, Eq. (1) has the triangular periodic solutions in the form:

$$\varphi(\eta) = \frac{\sqrt{ab}}{b}\tan[\sqrt{ab}(\eta+\eta_0)] \tag{3}$$

$$\varphi(\eta) = -\frac{\sqrt{ab}}{b}\cot[\sqrt{ab}(\eta+\eta_0)] \tag{4}$$

where η_0 is the integration constant.

Case B: Suppose that $ab < 0$, Eq. (1) has the soliton solutions in the form:

$$\varphi(\eta) = -\frac{\sqrt{-ab}}{b}\tan h[\sqrt{-ab}(\eta+\eta_0)] \tag{5}$$

$$\varphi(\eta) = -\frac{\sqrt{-ab}}{b}\cot h[\sqrt{-ab}(\eta+\eta_0)] \tag{6}$$

where η_0 is the integration constant, tanh and coth are the hyperbolic tangent and hyperbolic cotangent function, respectively.

3. Analytical solutions to the GNLSE (1)

Assume that Eq. (1) admits exact traveling wave solutions in the form:

$$u(\xi,\tau) = A(\eta)e^{i(k\xi - c\tau)}, \quad \eta = K\xi - v\tau \tag{7}$$

Substituting $u(\xi, \tau)$ into Eq. (1), and letting the real part and imaginary part be zero, we get

$$(6\beta_3 - \beta_4 c)v^3A''' + (6K - 6\beta_2 cv + \beta_4 c^3 v - 18\beta_3 c^2 v)A' - 18svA^2A' + 3\Gamma A = 0 \tag{8}$$

$$\beta_4 v^4 A'''' - 48v\tau_R A^2 A' + 6v^2(2\beta_2 - \beta_4 c^2 + 12\beta_3 c)A'' - 24(\gamma_1 + cs)A^3 - 24\gamma_2 A^5 + (24k - 12\beta_2 c^2 + \beta_4 c^4 - 24\beta_3 c^3)A = 0 \tag{9}$$

where K, v, k and c are real constants, $A(\eta)$ is a real function of η, and A', A'', A''' and A'''' represent $dA/d\eta$, $d^2A/d\eta^2$, $d^3A/d\eta^3$ and $d^4A/d\eta^4$, respectively.

Differentiating Eq. (8) with respect to η once, we have

$$(6\beta_3 - \beta_4 c)v^3A'''' = -(6K - 6\beta_2 cv - 18\beta_3 c^2 v + \beta_4 c^3 v)A'' + 18svA^2A'' + 36svAA'^2 - 3\Gamma A' \tag{10}$$

Substituting A'''' into Eq. (9), we obtain

$$\begin{aligned} &v[6v(6\beta_3 - \beta_4 c)(2\beta_2 - \beta_4 c^2 + 12\beta_3 c) - \beta_4(6K - 6\beta_2 cv - 18\beta_3 c^2 v + \beta_4 c^3 v)] \times A'' + 18s\beta_4 v^2 A^2 A'' \\ &\quad + 36s\beta_4 v^2 AA'^2 - 3\Gamma\beta_4 vA' - 48v\tau_R(6\beta_3 - \beta_4 c)A^2A' + (24k - 12\beta_2 c^2 + \beta_4 c^4 - 24\beta_3 c^3)(6\beta_3 - \beta_4 c)A \\ &\quad - 24(\gamma_1 + cs)(6\beta_3 - \beta_4 c)A^3 - 24\gamma_2(6\beta_3 - \beta_4 c)A^5 = 0 \end{aligned} \tag{11}$$

Now, we use sub-ODE expansion method to construct analytical solutions of the Eq. (11), the sub-ODE and its solutions are given in Section 2. According to the homogeneous balance method, we find Eq. (6) admits the following form solution:

$$A(\eta) = a_0 + a_1\varphi(\eta), \quad a_1 \neq 0 \tag{12}$$

where a_0 and a_1 are real constants to be determined later.

Inserting (2) and (12) into Eq. (11), we have

$$\begin{aligned} &a_1 v[6v(6\beta_3 - \beta_4 c)(2\beta_2 - \beta_4 c^2 + 12\beta_3 c) - \beta_4(6K - 6\beta_2 cv - 18\beta_3 c^2 v + \beta_4 c^3 v)] \times (2ab\varphi + 2b^2\varphi^3) \\ &\quad + 18s\beta_4 v^2 a_1(a_0 + a_1\varphi)^2(2ab\varphi + 2b^2\varphi^3) + (24k - 12\beta_2 c^2 + \beta_4 c^4 - 24\beta_3 c^3)(6\beta_3 - \beta_4 c)(a_0 + a_1\varphi) \\ &\quad + 36s\beta_4 v^2 a_1^2(a_0 + a_1\varphi)(a^2 + 2ab\varphi^2 + b^2\varphi^4) - 3\Gamma\beta_4 va_1(a + b\varphi^2) - 48v\tau_R a_1 \times (6\beta_3 - \beta_4 c)(a_0 + a_1\varphi)^2(a + b\varphi^2) \\ &\quad - 24(\gamma_1 + cs)(6\beta_3 - \beta_4 c)(a_0 + a_1\varphi)^3 - 24\gamma_2(6\beta_3 - \beta_4 c)(a_0 + a_1\varphi)^5 = 0 \end{aligned} \tag{13}$$

Gathering all terms with the same order of φ, and making the parameter of each order of φ equal to zero, thus, a series of algebraic equations are got

$$\begin{aligned} &\varphi^0 : (24k - 12\beta_2 c^2 + \beta_4 c^4 - 24\beta_3 c^3)(6\beta_3 - \beta_4 c)a_0 + 36s\beta_4 v^2 a_0 a_1^2 a^2 - 3\Gamma\beta_4 va_1 a - 48v\tau_R a_0^2 a_1 a(6\beta_3 - \beta_4 c) \\ &\quad - 24a_0^3(\gamma_1 + cs)(6\beta_3 - \beta_4 c) - 24\gamma_2 a_0^5(6\beta_3 - \beta_4 c) = 0 \end{aligned} \tag{14}$$