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Shift operator method and Runge-Kutta exponential time differencing method for plasma

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ABSTRACT

In this paper, the shift operator finite difference time domain (SO-FDTD) method and Runge-Kutta exponential time differencing (RKETD) method are introduced. The high accuracy and efficiency of the two methods are verified by calculating the reflection and transmission coefficients of electromagnetic waves through a collisional plasma slab. A comparison of computational efficiency of the two methods is presented by simulating the electromagnetic wave propagation in homogeneous non-magnetized plasma. The numerical results indicate that the calculation time using SO-FDTD method is less than that using RKETD method with almost the same accuracy.

1. Introduction

Over the last decade, there have been numerous investigations of finite difference time domain (FDTD) formulations on dispersive medium. The methods employed include the recursive convolution (RC) methods [1], the auxiliary differential equation (ADE) methods [2], frequency-dependent Z transform methods [3], piecewise linear recursive convolution (PLRC) method [4], piecewise linear current density recursive convolution (PLCDRC) method [5–7], shift operator (SO-FDTD) method [8,9], exponential time differencing (ETD) method [10,11] and Runge-Kutta exponential time differencing (RKETD) method [12].

In this paper, the SO-FDTD and the second-order RKETD schemes are derived for transient propagation in plasma medium. The algorithms are confirmed by comparing the reflection and transmission coefficients for a non-magnetized collisional plasma slab with exact result. The two methods are both applied to simulate the electromagnetic wave propagation in non-magnetized collisional plasma slab. A comparison, with respect to the computational efficiency, between the two methods for electromagnetic waves is presented. It is concluded that the SO-FDTD method is more efficient than the RKETD method, in the sense that it uses less memory storage in the computing processes, although the two methods both present similar accuracy and mathematical efficiency.

2. Methods of homogeneous non-magnetized plasma

2.1. Shift operator (SO-FDTD) method [8,9]

With collisional cold plasma in dispersive medium, the well-known Maxwell's equations and related equations are given as follows:

$$\frac{\partial D}{\partial t} = \nabla \times H, \tag{1}$$

$$\frac{\partial H}{\partial t} = -\frac{1}{\mu_0}\nabla \times E, \tag{2}$$

$$D(\omega) = \varepsilon_0 \varepsilon_r(\omega) E(\omega). \tag{3}$$

The dielectric constant $\varepsilon_r(\omega)$ can be written as rational fractional function:

$$\varepsilon_r(\omega) = \frac{\sum_{n=0}^{N} p_n (j\omega)^n}{\sum_{n=0}^{N} q_n (j\omega)^n}. \tag{4}$$

According to SO-FDTD method [8,9], the recurrence relations of electric field **E** can be written as

$$E^{n+1} = \frac{1}{b_0}\left[a_0\left(\frac{D^{n+1}}{\varepsilon_0}\right) + a_1\left(\frac{D^n}{\varepsilon_0}\right) + a_2\left(\frac{D^{n-1}}{\varepsilon_0}\right) - b_1 E^n - b_2 E^{n-1}\right], \tag{5}$$

where $a_0 = q_0 + q_1\frac{2}{\Delta t} + q_2\left(\frac{2}{\Delta t}\right)^2$; $a_1 = 2q_0 - 2q_2\left(\frac{2}{\Delta t}\right)^2$; $a_2 = q_0 - q_1\frac{2}{\Delta t} + q_2\left(\frac{2}{\Delta t}\right)^2$; $b_0 = p_0 + p_1\frac{2}{\Delta t} + p_2\left(\frac{2}{\Delta t}\right)^2$; $b_1 = 2p_0 - 2p_2\left(\frac{2}{\Delta t}\right)^2$; $b_2 = p_0 - p_1\frac{2}{\Delta t} + p_2\left(\frac{2}{\Delta t}\right)^2$.

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As to cold plasma, relative dielectric constant is:

$$\varepsilon_r(\omega) = 1 + \frac{\omega_p^2}{\omega(jv_c - \omega)}. \tag{6}$$

Where ω_p and v_c are the plasma frequency and electron collision frequency, respectively, comparing Eq. (6) with Eq. (4) we have $N = 2$ and $p_0 = \omega_p^2, \quad p_1 = v_c, \quad p_2 = 1, \quad q_0 = 0, \quad q_1 = v_c, \quad q_2 = 1$.

According to Eq. (5), the **E** can be calculated. The iterative equation of magnetic field is similar to that of the conventional FDTD.

2.2. Runge-Kutta exponential time differencing (RKETD) method [12]

Considering an isotropic collisional plasma medium. The Maxwell's equations and constitutive relation for non-magnetized collision plasma are given by

$$\nabla \times H = \varepsilon_0 \frac{\partial E}{\partial t} + J, \tag{7}$$

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}, \tag{8}$$

$$\frac{dJ}{dt} + v_c J = \varepsilon_0 \omega_p^2 E. \tag{9}$$

Multiplying Eq. (9) by $e^{v_c t}$ and integrating the equation over a single step from t_n to t_{n+1} [12], after some integral manipulation the component of J at $n+1$ time step can be written as

$$J_i^{n+1} = e^{-v_c \Delta t} J_i^n + \frac{1 - e^{-v_c \Delta t}}{v_c} \varepsilon_0 \omega_p^2 E_i^n + \frac{e^{-v_c \Delta t} - 1 + v_c \Delta t}{v_c^2 \Delta t} [\varepsilon_0 \omega_p^2 E_i^{n+1} - \varepsilon_0 \omega_p^2 E_i^n], \tag{10}$$

where $i = x, y, z$.

Using the Yee grid and leap-frog integration, Eq. (7) can be expressed as

$$E_i^{n+1} = E_i^n + \frac{\Delta t}{\varepsilon_0} (\tilde{\nabla} \times H)_i^{n+1/2} - \frac{\Delta t}{2\varepsilon_0} (J_i^{n+1} + J_i^n). \tag{11}$$

here, $\tilde{\nabla}$ is discrete curl operator.

Combining Eq. (10) and Eq. (11), we have

$$E_i^{n+1} = \frac{1}{1 + (\omega_p^2 / 2v_c^2)(e^{-v_c \Delta t} - 1 + v_c \Delta t)} \times \left[1 - \frac{\Delta t \omega_p^2}{2 v_c} (1 - e^{-v_c \Delta t}) + \frac{\omega_p^2}{2 v_c^2} (e^{-v_c \Delta t} - 1 + v_c \Delta t) \right] E_i^n + \frac{\Delta t}{\varepsilon_0} (\tilde{\nabla} \times \mathbf{H})_i^{n+1/2} - \frac{\Delta t}{2\varepsilon_0} (1 + e^{-v_c \Delta t}) J_i^n. \tag{12}$$

The update equation of **H** field is given by

$$H_i^{n+1/2} = H_i^{n-1/2} - \frac{\Delta t}{\mu_0} (\tilde{\nabla} \times \mathbf{E})_i^n \tag{13}$$

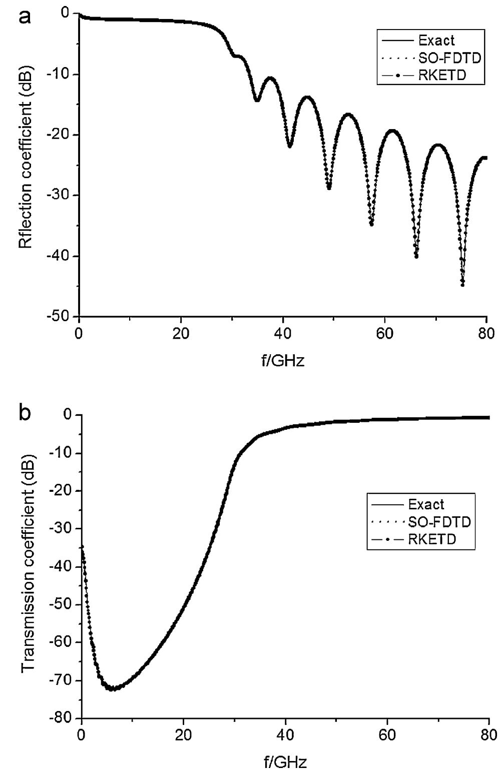

Fig. 1. Comparison of the reflection and transmission coefficients for plasma by exact solution, SO-FDTD and RKETD. (a) Reflection coefficient. (b) Transmission coefficient.

3. Validity and precision of SO-FDTD method and RKETD method

In order to check the correctness and the accuracy of upper algorithm, we compute the reflection and transmission coefficients of electromagnetic wave through a non-magnetized collision plasma slab with a thickness of 1.5 cm. The plasma parameters are as follows: $\omega_p = 2\pi \times 28.7 \times 10^9$ rad/s and $v_c = 20 \times 10^9$ Hz. The incident wave used in the simulation is the derivative of the Gaussian pulse. The computational domain is subdivided into 1000 cells, the plasma occupies 200 cells in the middle, and other cells are free space. The space-step is 75 μm and time-step is 0.125 ps. The reflection and transmission coefficients are computed from the reflected and transmitted pulses through discrete Fourier transform of the time-dependent electric field components. Fig. 1 indicates that the simulation results agree with exact values very well. The accuracy of RKETD method is similar to that of SO-FDTD method.

4. The comparison of computational efficiency

Now, we perform a simulation of electromagnetic wave hitting a non-magnetized plasma slab, which uses the properties of

Table 1
The time compare statistics and efficiency analysis between SO-FDTD method and RKETD method.

Frequency (GHz)	Time steps	RKETD calculation time (s)	SO-FDTD calculation time (s)	Saving times (s)	Percentage of saving time (s)
10	500	0.406	0.360	0.046	11.33%
	830	0.672	0.594	0.078	11.61%
	1200	0.969	0.859	0.110	11.35%
60	500	0.407	0.359	0.048	11.79%
	1100	0.906	0.797	0.109	12.03%
	1450	1.172	1.031	0.141	12.03%