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A new chaotic system without linear term and its impulsive synchronization

Yuhua Xu[a,b], Yuling Wang[c,*]

[a] Department of Maths, Yunyang Teachers' College, Hubei, Shiyan 442000, PR China
[b] Computer School of Wuhan University, Wuhan 430079, PR China
[c] School of Economics, South-Central University for Nationalities, Wuhan 430074, PR China

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ABSTRACT

In this paper, a new chaotic system without linear term is presented by ordinary differential equations, and some basic dynamical properties are studied. Moreover, the impulsive synchronization problem of the new chaotic systems is investigated by using uncertain impulsive control matrix. Using the impulsive theory, sufficient conditions are derived for the synchronization of new chaotic systems. Numerical simulation example is provided to verify the effectiveness of the proposed approach.

1. Introduction

Since Lorenz found the first chaotic attractor, considerable research interests have been made in searching for new chaotic attractors. Particular, research interests are turning in searching for new chaotic attractors in the three-dimensional (3D) autonomous ordinary differential equations. For example, Lorenz system [1], Rössler system [2], Chen system [3], Lü system [4] and Liu system et al. [5] were reported and analyzed. In very recent years, creating complex multiscroll or multi-wing chaotic attractors in 3D autonomous systems has been rapid developed [6–9]. Most chaotic attractors in 3D autonomous systems contain the linear term by the existing literatures. Furthermore, Pecora and Carroll [10] first proposed drive-response method to achieve synchronization between two chaotic systems since 1990, and then other methods have been proposed such as the sliding mode control [11], observer-based control [12–15], backstepping control [16], nonlinear control [17–21], and so on.

So far chaotic attractors without linear term in three ODEs have seldom been found, whether the 3D autonomous systems without linear term can generate chaos, these problems have rarely been investigated. Moreover, the impulsive control matrix is always certain matrix in the existing literature on impulsive control method. In this paper, a new chaotic system without linear term is constructed by ordinary differential equations, the basic dynamic behaviors of the system were studied, and impulsive synchronization was achieved by using uncertain impulsive control matrix.

2. New chaotic system

Consider the following four-term simple system

$$\begin{cases} \dot{x} = ln(a + e^{y-x}), \\ \dot{y} = xz, \\ \dot{z} = b - xy, \end{cases} \tag{1}$$

which is chaotic for the parameters $a = 0.1$, $b = 0.25$, e is a transcendental number (see Figs. 1–4).

Remark 1. If using the MacLaurin formula, $\ln(a + e^{y-x})$ may have infinite terms in the expression form.

When the parameters $a = 0$, $b = 0.25$, the system (1) change into the following five-term simple system

$$\begin{cases} \dot{x} = y - x, \\ \dot{y} = xz, \\ \dot{z} = b - xy, \end{cases} \tag{2}$$

which is not chaotic for the parameters $a = 0$, $b = 0.25$ (see Fig. 5).

3. Some basic properties of the simple chaotic system (1)

In this section, we will investigate some basic properties of the new system (1).

* Corresponding author. Fax: +86 7198846038.
E-mail address: yhwndb@163.com (Y. Wang).

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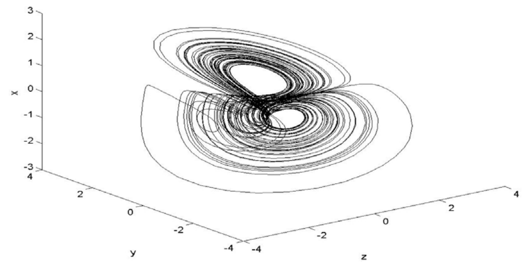

Fig. 1. A new four-term simple chaotic attractor.

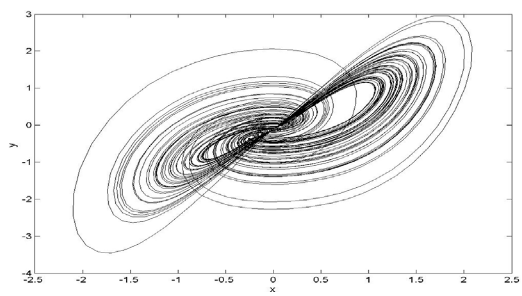

Fig. 2. $x-y$ phase plane the four-term chaotic attractor.

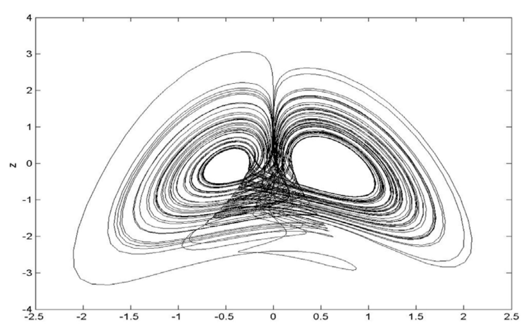

Fig. 3. $x-z$ phase plane the four-term chaotic attractor.

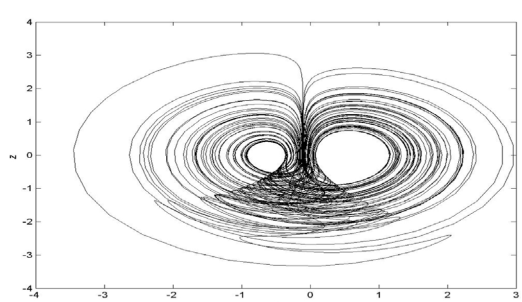

Fig. 4. $y-z$ phase plane the four-term chaotic attractor.

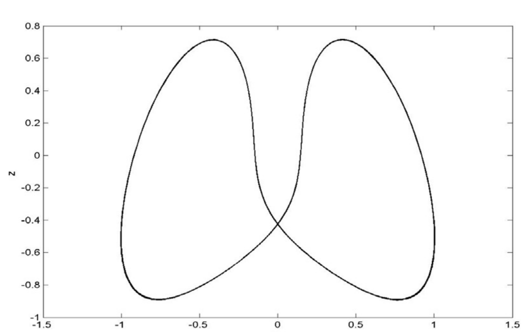

Fig. 5. $x-z$ phase plane the system (2).

3.1. Equilibria

Let

$$\begin{cases} \ln(a+e^{y-x})=0, \\ xz=0, \\ b-xy=0, \end{cases} \tag{3}$$

when $a<1$, $\ln^2(1-a)+4b\geq 0$, the system has two equilibrium points, respectively, which are described as follows

$$E_1=\left(\frac{-\ln(1-a)-\sqrt{\ln^2(1-a)+4b}}{2},\frac{\ln(1-a)-\sqrt{\ln^2(1-a)+4b}}{2},0\right),$$

$$E_2=\left(\frac{-\ln(1-a)-\sqrt{\ln^2(1-a)+4b}}{2},\frac{\ln(1-a)-\sqrt{\ln^2(1-a)+4b}}{2},0\right),$$

Let the parameters $a=0.1$ and $b=0.25$. So, the system equilibrium points can be easily obtained as

$E_1=(-0.4976, -0.6029, 0)$, $E_2=(0.6029, 0.4976, 0)$. For equilibrium $E_1=(-0.4976, -0.6029, 0)$, the system (3) is linearized, and the Jacobian matrix is defined as

$$J=\begin{pmatrix} -\dfrac{e^{y-x}}{a+e^{y-x}} & \dfrac{e^{y-x}}{a+e^{y-x}} & 0 \\ z & 0 & x \\ -y & -x & 0 \end{pmatrix}_{E_1}$$

To gain its eigenvalues, we let $\left|\lambda I-J\right|=0$.

So the corresponding eigenvalues at $E_1=(-0.4976, -0.6029, 0)$, are

$$\lambda_1=0.1299,\ \lambda_2=-0.5489+0.2983i,\ \lambda_3=-0.5489-0.2983i \tag{4}$$

In addition, the corresponding eigenvalues at $E_2=(0.6029, 0.4976, 0)$ are

$$\lambda_1=-1.0772,\ \lambda_2=0.0886+0.7393i,\ \lambda_3=0.0886-0.7393i \tag{5}$$

From Eqs. (4) and (5), E_1, E_2 are all saddle focus-nodes. So these equilibria point are unstable.

3.2. Dissipativity and the existence of attractor

For dynamical system (1), we can obtain

$$\Delta V=\frac{\partial\dot{x}}{\partial x}+\frac{\partial\dot{y}}{\partial y}+\frac{\partial\dot{z}}{\partial z}=-\frac{e^{y-x}}{0.1+e^{y-x}},$$

where $\frac{e^{y-x}}{0.1+e^{y-x}}$ is a negative value. Dynamical system (1) is one dissipative system, and an exponential contraction of the system (1) is $\frac{dV}{dt}=e^{-\frac{e^{y-x}}{0.1+e^{y-x}}}$.

In the dynamical system (1), a volume element V_0 is apparently contracted by the flow into a volume element $V_0e^{-\frac{e^{y-x}}{0.1+e^{y-x}}}$ in time t. It means that each volume containing the trajectory of this dynamical system shrinks to zero as $t\to\infty$ at an exponential rate $-\frac{e^{y-x}}{0.1+e^{y-x}}$. So, all this dynamical system orbits are eventually confined to a specific subset that have zero volume, the asymptotic motion settles onto an attractor of the system (1).

3.3. Lyapunov exponent

Any system containing at least one positive Lyapunov exponent is defined to be chaotic. The Lyapunov exponent spectrum of the system (1) is found to be $L_1=0.1139$, $L_2=0.0347$, $L_3=-0.2970$ for