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Analysis and demonstration of multiplexed phase computer-generated hologram for modal wavefront sensing

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ABSTRACT

We propose to implement a modal wavefront sensor (MWS) using a multiplexed phase computer-generated hologram (MPCGH). Based on general orthogonal aberration modes, the theoretical treatments of the MWS employing a MPCGH are presented with scalar diffraction approximations and Fourier analysis. Under the small aberration approximations, we give the analytical formula for characterizing the relationship between the sensor signal and the amplitude of the aberration mode. We design several MPCGHs with an effective method of modified off-axis reference beam holograms superposition, and code some common orthogonal Zernike aberration modes into the MPCGH. The numerical simulation is carried out to investigate the performance of MWS to detect particular aberration mode(s). The results exhibit the expected responses of the corresponding symmetric spot pair, and indicate that the wavefront distorted by a special Zernike aberration mode, after modulated by the MPCGH, can be transformed into beams with an intensity-normalized differential signal, which can reflect the change trend of the aberration coefficients in the test wavefront. The experimental demonstration with designed MPCGHs in conjunction with two phase-only spatial light modulators was carried out to test the performance of the MWS.

1. Introduction

A laser beam can be seriously degraded due to high turbulence when transmitting through the boundary layer as well as the atmosphere between the platform and the target in airborne environments, and the use of wavefront sensors for ultrafast adaptive optics are needed. Wavefront sensor can be utilized to detect the unknown aberrations and supply driven signals for the actuators of the corrective elements such as deformable mirrors. Many wavefront sensors have been proposed and applied to adaptive optical system, such as shearing interferometers, Shack–Hartmann [1], curvature [2] and pyramid wavefront sensors [3], each with their own virtues and shortcomings [4]. Ghebremichael et al. proposed a holographic modal wavefront sensor (HMWS) with multiplexed holographic optical elements (MHOEs) [5]. This sensor needs no heavy computation with high detecting precision, small errors, and fast speed. However, the generation of MHOEs may be the key obstacle to bring the sensor into practice, because the optical recording of MHOEs and cross-talks between certain modes are both troublesome problems. The multiple computer-generated hologram (MCGH) is a better alternative to implement specific MHOEs, for it provides the design-freedom to optimize the cross-talks. An approach for HMWS using a MCGH was proposed by Bhatt et al., and some simulation results were presented to validate this technology schematically [6]. But it seems so unrealistic that, in order to measure the arbitrary amplitude of a special aberration mode, one has to add together an infinite number of subholograms. Ghebremichael and Mishral reduced the necessary number of precoded subhologram to two per aberration mode, taking advantage of the intramodal crosstalk effect. The drawback of their approaches are that, in terms of computer-generated hologram, with convergent spherical waves as the reference waves to record the subholograms, it is unavoidable for the reconstructed focal pattern to be disturbed by the conjugate divergent spherical waves. To generate an effective MCGH, we have proposed to employ the binary MPCGH, because of their relative high efficiency, for the implementation of HMWS, with less number of subholograms, and shown the simulation results [7].

In Section 2, we present further detailed theoretical treatments to the HMWS based on the MPCGH, with scalar diffraction approximations and Fourier analysis. The relationship formula

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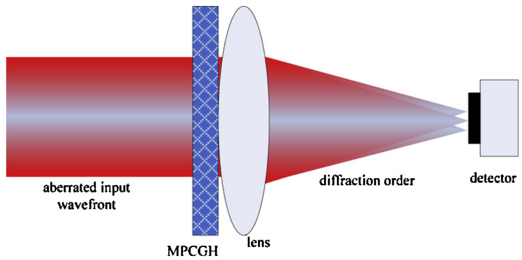

Fig. 1. Schematic diagram of the modal wavefront sensor using a MPCGH. An aberrated input light beam illuminates the assembly of the MPCGH and Fourier-transforming lens, and then it is received by a detector.

between the sensor signal and the amplitude of aberration mode is derived under the condition of small aberration approximation. The corresponding numerical analysis results, in addition to the design of several particular MPCGHs, are provided and discussed in Section 3. In Section 4, we report the experimental realization of the HMWS based on the MPCGHs with the aid of two phase-only spatial light modulators (SLMs) and in Section 5 we present the conclusion and discussion.

2. Theoretical treatment and analysis

The aberrated wavefront can be described by decomposition into a set of different modes using orthogonal polynomials. The amplitude of each mode can be figured out through a proper measurement approach, and thus the full reconstruction of wavefront can be obtained. The schematic configuration of the built HMWS is depicted in Fig. 1. First, the incoming light beam with an aberrated wavefront illuminates a transparent, phase MHOE. Afterwards the light passes a Fourier lens, and finally reaches a detector (usually a Charge Coupled Device, CCD camera). The camera captures the focal diffraction pattern and allows digital processing of the data. The key component in the holographic modal wavefront sensor system locates the MHOE. Multiple aberration modes have been coded into the element. We will present the generation and reconstruction process of MHOE to illustrate the operation principle of the MWS in the following.

Suppose we sequentially code N aberration modes into the same area of transparent material by modulating refractive index, and yield the complex transmittance as

$$t_N = \exp\left[j\frac{m}{N}\sum_k \cos(\alpha_k\varphi_k + \tau_k)\right], \tag{1}$$

where $\{\varphi_k\}, \quad \{\alpha_k\}, \quad \{\tau_k\}, \quad k = 1, \ldots, N$ are, respectively, the aberration modes to be coded, modes' coefficients and corresponding tilt factors of the reference waves, with m the modulation depth and $j = \sqrt{-1}$ the imaginary constant. Here we introduce the normalization factor $1/N$ so as to ensure modulation range varying between $\pm m$. With the following expansion of exponential in terms of Bessel function,

$$\exp(jz\cos\phi) = J_0(z) + 2\sum_{s=1}^{\infty} j^s J_s(z)\cos s\phi, \tag{2}$$

i.e., the nominal Jacobi's identity, we yield an alternative form of t_N with continued products as

$$t_N = \prod_k \left\{ J_0\left(\frac{m}{N}\right) + \sum_{s=1}^{\infty} j^s J_s\left(\frac{m}{N}\right)(\exp[js(\alpha_k\varphi_k + \tau_k)] + \exp[-js(\alpha_k\varphi_k + \tau_k)]) \right\}, \tag{3}$$

where $J_s(\cdot)$is the sth order Bessel function of the first kind.

Provided that the unknown wavefront comprises a single aberration mode, i.e., $U = \exp(j\beta_q\varphi_q)$, in which β_q is the coefficient of the qth order aberration mode φ_q, with the modulation and transformation by the assembly of multiplexed holographic element and Fourier lens, thus we yield the complex-amplitude in the focal plane

$$U_d = \mathrm{FT}\{U\} * \left(\prod_k (*) \left[J_0\left(\frac{m}{N}\right)\delta(\mathbf{f}) + \sum_{s=1}^{\infty} j^s J_s\left(\frac{m}{N}\right)(V^+_{s,k} + V^-_{s,k}) \right] \right), \tag{4}$$

where

$$V^{\pm}_{s,k} = \mathrm{FT}\{\exp(\pm js\alpha_k\varphi_k)\} * \delta(\mathbf{f} \mp s\tilde{\boldsymbol{\tau}}_k), \tag{5}$$

$\prod_k(*)$ denotes continued convolution operation, $\mathrm{FT}\{\cdots\}$ indicates Fourier transformation, with $\delta(\cdot)$ Dirc-delta, $\mathbf{f} = (f_x, f_y)$ is the spatial frequency vector, $\tilde{\boldsymbol{\tau}}_k = (\nabla\tau_k)/2\pi$ is the spatial frequency coordinate corresponding to the kth tilt reference wave, and $\triangledown$ is the gradient operator, and $s\tilde{\boldsymbol{\tau}}_k$ represents different shift-distance from the center in the detector plane. Representation (4) has a truly cumbersome form, fortunately we only need to concentrate on the first term of the infinite summation due to the limit size of sensitive area of CCD camera. Here, suppose the CCD camera utilized has a sufficient small sensitive area, and then the higher diffraction orders in the detector plane might be isolated, only lower ones remain. For simplicity, we give an illustrative example with a low number of aberration modes so that the reader may more easily understand the meaning of the above expression. Suppose that we code two aberration modes, i.e., $N = 2$, and fixate on the complex-amplitude by taking $s = 1$, yielding

$$U^1_d = \left[J_0\left(\frac{m}{2}\right)\right]^2 \Upsilon_{zero} + jJ_0\left(\frac{m}{2}\right)J_1\left(\frac{m}{2}\right)\Upsilon_{linear} - \left[J_1\left(\frac{m}{2}\right)\right]^2 \Upsilon_{cross}, \tag{6}$$

where

$$\Upsilon_{zero} = \mathrm{FT}\{U\}\delta(\mathbf{f}) = \mathrm{FT}\{\exp(j\beta_q\varphi_q)\}\delta(\mathbf{f}), \tag{7}$$

$$\begin{aligned}\Upsilon_{linear} = {} & \mathrm{FT}\{\exp[j(\alpha_1\varphi_1 + \beta_q\varphi_q)]\} * \delta(\mathbf{f} - \tilde{\boldsymbol{\tau}}_1) \\ & + \mathrm{FT}\{\exp[-j(\alpha_1\varphi_1 - \beta_q\varphi_q)]\} * \delta(\mathbf{f} + \tilde{\boldsymbol{\tau}}_1) \\ & + \mathrm{FT}\{\exp[j(\alpha_2\varphi_2 + \beta_q\varphi_q)]\} * \delta(\mathbf{f} - \tilde{\boldsymbol{\tau}}_2) \\ & + \mathrm{FT}\{\exp[-j(\alpha_2\varphi_2 + \beta_q\varphi_q)]\} * \delta(\mathbf{f} + \tilde{\boldsymbol{\tau}}_2),\end{aligned} \tag{8}$$

$$\begin{aligned}\Upsilon_{cross} = {} & \mathrm{FT}\{\exp[j(\alpha_1\varphi_1 + \alpha_2\varphi_2 + \beta_q\varphi_q)]\} * \delta[\mathbf{f} - (\tilde{\boldsymbol{\tau}}_1 + \tilde{\boldsymbol{\tau}}_2)] \\ & + \mathrm{FT}\{\exp[j(\alpha_1\varphi_1 + \alpha_2\varphi_2 + \beta_q\varphi_q)]\} * \delta[\mathbf{f} - (\tilde{\boldsymbol{\tau}}_1 + \tilde{\boldsymbol{\tau}}_2)] \\ & + \mathrm{FT}\{\exp[-j(\alpha_1\varphi_1 - \alpha_2\varphi_2 - \beta_q\varphi_q)]\} * \delta[\mathbf{f} + (\tilde{\boldsymbol{\tau}}_1 - \tilde{\boldsymbol{\tau}}_2)] \\ & + \mathrm{FT}\{\exp[j(\alpha_1\varphi_1 - \alpha_2\varphi_2 + \beta_q\varphi_q)]\} * \delta[\mathbf{f} - (\tilde{\boldsymbol{\tau}}_1 - \tilde{\boldsymbol{\tau}}_2)].\end{aligned} \tag{9}$$

Representation (6) analytically gives the complex field distribution comprising several first diffraction orders and zero order. Here $\Upsilon_{zero}, \quad \Upsilon_{linear}, \quad \Upsilon_{cross}$ denote, respectively, the zeroth, first-linear and first-cross diffraction orders in terms of the coded aberration