Optik 125 (2014) 2652–2655

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Elimination of zero-order and conjugate images in off-axis digital holography

Peizhen Qiu*, Juguan Gu

School of Science, Huzhou Teachers College, Huzhou, 313000, PR China

ARTICLE INFO

Article history:
Received 4 June 2013
Accepted 2 November 2013

Keywords:
Off-axis digital hologram
Filtering technique
The zero-order and conjugate image

ABSTRACT

In this paper, a new digital method has been developed for eliminating the zero-order and conjugate image in off-axis digital holography. The method consists of filtering the spatial frequencies associated with these undesired terms (the zero-order and conjugate images) in the computed Fourier transform plane of the hologram multiplied with computer simulated original reference wave (written as (RI_h)). An important conclusion is that, without the need for additional shift-spectrum process, the spectrum of real image always appears in the center of the spectrum plane and never shifts. This brings us some convenient to design a filtering window. The theoretical analysis and the corresponding simulation and experiment results are given. As the zero-order and conjugate image are eliminated, both the contrast and brightness of the real image are enhanced and the image quality is improved. Meanwhile, the process of designing a filtering window is simple and convenient.

1. Introduction

With the progress of computer and CCD technology, the digital holography (DH) using a CCD instead of photosensitive materials is becoming a hot research area [1–6]. Compared with classical holography, the main advantage of DH is that both the amplitude and phase information of the recorded object can be achieved directly in numerical reconstruction process and the chemical processing can be avoided. So, the DH has been widely applied in many fields such as shape measurement [2,3], microscopy [4], measurement of size and position of particles [5].

Off-axis recording geometry [7] generally used in DH, and its reconstructed image contains three parts: the real image of the object, zero-order image and conjugate image. In this case, even if the three parts appear at different locations in the reconstructed images, the elimination of the undesired terms is interesting, because the existence of the zero-order image and conjugate image limit the size of the real image area and adversely affect the image quality. The methods proposed to eliminate the zero-order and conjugate image fall into four main categories: (i) the phase shifting holography technique; (ii) spatial domain filtering technique; (iii) frequency spectrum plane filtering technique; and (iv) image plane filtering technique.

The phase shifting holography technique [8] needs using a robust optical system in a laboratory by changing the phase of reference wave and recording multiple holograms. And then through the mathematical process, only the real image is obtained.

The spatial domain filtering technique [9] is based on only one hologram and filters the hologram directly in spatial domain by using a filter.

Ref. [10] proposed that the zero-order and conjugate image can be digitally eliminated by means of filtering their associated frequency spectrum in the computed Fourier transform of the hologram, free of phase modulator or other extra equipment.

In Ref. [11], a filter window is set in the image plane to extract the image of the object field, and then the object field reached hologram plane is formed using diffraction's inverse operation. At last, the object field is reconstructed through diffraction's angular spectrum theory.

In Ref. [12], by adding the extra reconstruction plane and the formularized numerically reference wave in the reconstruction procedure, the blurring images (the zero-order and conjugate image) can be separated out and removed from the reconstructed image. Then, by performing the back propagating to the exact reconstruction plane, the pure object image without the blurring can be obtained.

However, to our best knowledge, in the present literature, whether frequency spectrum plane filtering technique or image plane filtering technique, the spectral position of the real image (or the position of the real image) locates on one side of the spectrum of the zero-order image (or zero-order image), and the spectrum of the zero-order image (or zero-order image) locates at the center of the spectrum plane (or image plane).

This brings some difficulties to design a suitable filtering window. In order to obtain accurately the frequency spectrum of the

* Corresponding author.
E-mail address: qiupeizhen@126.com (P. Qiu).

0030-4026/$ – see front matter © 2013 Elsevier GmbH. All rights reserved.
http://dx.doi.org/10.1016/j.ijleo.2013.11.022

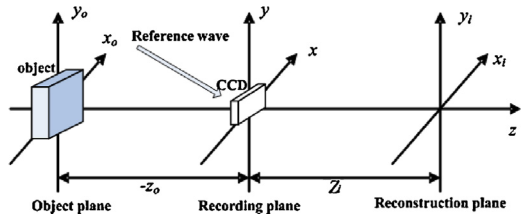

Fig. 1. Simplified optical path of off-axis digital holography and its coordinate definition.

real image (or the real image itself), the process of designing a filtering window must fully consider both the position and size of the spectrum of the real image (or the real image). And another problem is, after filtering, the real image appears at the side of the reconstructed image plane.

In response to these problems, we propose a new method to eliminate the zero-order and conjugate image. The method can be summarized as: firstly, multiply digital hologram with computer simulated original reference wave (written as (RI_h)). Secondly, use (RI_h) to do fast Fourier transform (FFT), and then the spectrum of (RI_h) is obtained. We find that, without the need for extra shift-spectrum process, the spectrum of real image always appears in the center of the spectrum plane and never shifts. But the corresponding spectrums of the zero-order and conjugate image are affected by the off-axis reference wave and shifted in spectrum plane. Afterward, multiply a low-pass filter with the spectrum of (RI_h), and then through the inverse Fourier transform operation, the "digital filtered hologram" [12] is obtained. Finally, use the digital filtered hologram with Fresnel diffraction theory, only the object wave with no disturbance is obtained.

2. Theoretical analysis

2.1. Spectrum characteristics of digital hologram multiplied with simulated reference wave

The simplified optical path of off-axis digital holography and its coordinate definition is shown in Fig. 1. (x_0, y_0), (x, y) and (x_i, y_i) denote the coordinate of object plane, CCD recording plane and reconstruction plane, respectively. Assuming that the reference wave $R(x, y)$ is plane wave parallel to the y–z plane, the angle between wave vector and the z-axis is θ. So the complex amplitude distribution of the reference wave on CCD recording plane can be written in the following form:

$$R(x, y) = A_R(x, y)\exp(ik_0 y), \tag{1}$$

where $A_R(x, y)$ represents the amplitude of reference wave and its value is generally set to unity ($A_R(x, y) = 1$), and wave vector $k_0 = 2\pi \sin\theta/\lambda$.

Meanwhile, assume that the complex amplitude distribution of the object wave on CCD recording plane is given by

$$O(x, y) = A_o(x, y)\exp[i\phi_o(x, y)], \tag{2}$$

where $A_o(x, y)$ represents the amplitude of object wave, $\phi_o(x, y)$ represents the phase of object wave. The intensity distribution on CCD recording plane can be expressed as

$$I_h(x, y) = \left|O(x, y) + R(x, y)\right|^2 = \left|O\right|^2 + \left|R\right|^2 + OR^* + O^*R, \tag{3}$$

where "*" denotes complex conjugation. The two square terms correspond to the intensities of respectively the object and reference wave, OR^* is the image term and O^*R is its complex conjugate, i.e. the conjugate image. Transform Eq. (3) with Fourier transformation, we can obtain the frequency spectrum distribution of hologram, as shown in Eq. (4)

$$\hat{I}_h(f_x, f_y) = \hat{O} \otimes \hat{O}^*(f_x, f_y) + A_R^2\delta(f_x, f_y) + A_R\hat{O}(f_x, f_y - k_0) + A_R\hat{O}^*(f_x, f_y + k_0), \tag{4}$$

where "$\hat{\ }$" represents Fourier transform operator, $\otimes$ represents the convolution operator, k_0 represents the frequency shift induced by the reference wave. The reference wave having a constant intensity, i.e. A_R is a constant value, its spectrum results in a Dirac function δ. The first and second terms on the right-hand side of Eq. (4) are the spatial frequency spectrum of the zero-order image, which are located in the center of the frequency spectrum plane. The third and fourth terms which symmetrically locate at the side of the frequency spectrum plane are the spectrums of real image and conjugated image, respectively, because they are modulated by the reference wave $R(x,y)$ and shifted during the Fourier transform operation to hologram (see Fig. 4(b)).

If we multiply the hologram with the computer simulated original reference wave (written as (RI_h)), the Eq. (3) can be written as

$$RI_h(x, y) = |O|^2R + |R|^2R + O|R|^2 + O^*(R)^2. \tag{5}$$

Transform Eq. (5) with Fourier transformation, the two dimensional Fourier spectrum of (RI_h) is obtained, as shown in Eq. (4)

$$(\hat{RI}_h)(f_x, f_y) = \hat{O} \otimes \hat{O}^* A_R\delta(f_x, f_y + k_0) + A_R^3\delta(f_x, f_y + k_0) + A_R^2\hat{O}(f_x, f_y) + A_R^2\hat{O}^*(f_x, f_y + 2k_0), \tag{6}$$

Obviously, on the right-hand side of Eq. (5), the third term $O|R|^2$ is not affected by $R(x,y)$, thus its Fourier transformation (see the third term of Eq. (6)) locates in the center of the spectrum plane of (RI_h) and never shifts (see Fig. 4(d)). The other three terms of Eq. (5) are modified by $R(x,y)$ or $R^2(x,y)$ and shifted in the spectrum plane of (RI_h). This is the most important conclusion of this paper, and provides a theoretical basis for simplifying filter design. And it will bring us convenience for filter design.

2.2. Principles of eliminating the zero-order and conjugate image

In order to accurately extract the spectrum of real image, designing a filtering window is very important. According to Eq. (4), we must not only pay attention to the size of the filtering window, but also concern about the position of the filtering window. However, according to the above analysis, we find that the spectrum of real image always appears in the center of the spectrum plane of (RI_h) and never shifts. Hence, only the size of the filtering window just need considered. In this paper, we adopt a low pass filter to select the region of the spectrum of real image.

And then the digital filtered hologram is obtained by the inverse Fourier transform operation. Finally, use the digital filtered hologram with Fresnel diffraction theory, the object wave with no zero-order image and conjugate image is obtained.

The whole process can be expressed by the following two formulas:

$$O(x, y) = IFFT\{FFT(RI_h) \times Filter\}, \tag{7}$$

$$O(x_i, y_i) = \iint O(x, y)\exp\left[ik_0 z_i + ik_0\frac{(x_i - x)^2 + (y_i - y)^2}{2z_i}\right] dxdy, \tag{8}$$