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# Circuit design based manipulation of decoherence and disentanglement

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ABSTRACT

In the process of quantum information transport, environment inevitably causes decoherence and disentanglement. It is effective to constitute a hybrid qubit system by taking advantages of different types of qubits to overcome the effects of decoherence and achieve quantum information transport. We find that energy relaxation exists in the process of information exchange bewteen the hybrid qubits. Combining this kind of energy relaxation with the decoherence effects from external environment, quantum information transport of non-disentangled effect can be achieved in phase damping channel if the exchange decay rate and decoherence time satisfy certain constraint relations. We discuss the scheme to achieve the constraint relations through combining specific quantum circuits.



## 1. Introduction

The core of quantum computer is using controllable quantum systems as the hardware for scientific computing, information processing and quantum controlling. To achieve quantum computing, we need single-qubit gate and two-qubits gate, such as CNOT (controlled-NOT). To achieve CNOT gate, coherent coupling between the two qubits should be achieved first in a certain way. However, we need to disconnect one qubit from the other when we execute single quantum gate operation for one of the two qubits, under which the controllable coupling between qubits is significant [1–5]. Especially, controllable coherent coupling between qubits is necessary to implement large-scale quantum computing, where the physical system that suitable for quantum computing should be controlled coherently, can be prepared to a particular state and integrated to a series of criterion on a large scale. However, almost all the existing physical systems can not simultaneously satisfy these requirements. For example, the spin ensemble qubits can be well coherently manipulated but its scalability is poor [6,7]. The solid-state qubit, such as superconducting qubit and semiconductor quantum dot is regarded as one of the most promising solid candidates for achieving quantum computer because of its advantages in controllability, low loss, and scalability [8,9]. However, it is susceptible to outside influence and the decoherence time is short. Therefore, it is a key issue to take effective countermeasures in practical quantum information technologies to maintain long-time quantum coherence required by quantum information processing in the superconducting qubits-containing quantum system. Taking into account the improvement of decoherence time, easy integration and controllable coupling, the hybrid qubits program using superconducting qubits as the core should be considered [10–13].

Quantum entanglement is an important quantum information resource. Quantum information adopt entangled states as quantum channel, such as Bell-state quantum cryptography, quantum dense coding and quantum teleportation states [14,15]. Undoubtedly, the hybrid qubit system will also be influenced by decoherence and disentanglement caused by external environment [16]. Moreover, the latest researches show in the process of quantum information exchange, the exchange energy relaxation (dissipation) whose reasons is difficult to confirm is more easily occurred in hybrid qubits due to the multi-level structure of solid-state qubits or the mismatching of two qubits in information exchange. Exchange energy relaxation would also destroy the quantum entanglement between qubits, making the information exchange between the control qubit and the storage qubit conduct ineffectively, which suggests that controlling the exchange energy relaxation between the control qubit and storage qubit is very important that can not be ignored. In this paper, we choose superconducting qubits as the control qubits coupling to the storage qubits that possess longer decoherence time and then constitutes a hybrid qubit system. We present a scheme for disentanglement suppression based on the dynamical evolution of entanglement.

## 2. Model

This paper studies a hybrid quantum system, using superconducting qubits as the control qubits (denoted by sq) and the spin

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ensemble qubits as the storage qubits. Compared to the control qubit, the storage qubit (denoted as mq) has longer decoherence time. Taking into account the interaction between the system and environment, the Hamiltonian of the entire system is,

$$H = H_S + H_B + H_{SB} + H_{sq-mq}, \tag{1}$$

where

$$\hat{H}_S = \frac{1}{2}\omega^{sq}\sigma_z^{sq} + \frac{1}{2}\omega^{mq}\sigma_z^{mq} + g(\sigma_+^{sq}\sigma_-^{mq} + \sigma_-^{sq}\sigma_+^{mq}), \tag{2}$$

$H_B$ is the thermal reservoir Hamiltonian, $H_{SB}$ is the Hamiltonian of the interaction between system and external environment, $H_{sq-mq}$ is the exchange energy dissipation between the control qubits and storage qubits. $g$ is the strength of the interaction between the control qubits and the storage qubits, $\sigma_j^z$ and $\sigma_j^{\pm}$ ($j$ = sq, mq) are the Pauli operators which respectively obey the commutative and anticommutation of algebraic relations of the Pauli operator.

In the Markov process, the time evolution of the reduced density matrix $\rho$ of the coupled quantum system satisfies the Lindblad quantum master Eq. [17]

$$\dot{\rho} = -\frac{i}{\hbar}[H_S, \rho] - L\rho, \tag{3}$$

where

$$(4) L^{(1)}\rho = \frac{\Gamma^{sq}}{4}\left[\hat{\sigma}_z^{sq}, \left[\hat{\sigma}_z^{sq}, \rho\right]\right] + \frac{\Gamma^{mq}}{4}\left[\hat{\sigma}_z^{mq}, \left[\hat{\sigma}_z^{mq}, \rho\right]\right],$$

$$L^{(2)}\rho = \frac{\Gamma^{sm}}{2}(\sigma_z^{sq}\sigma_z^{mq}\rho - \sigma_z^{mq}\rho\sigma_z^{sq} + \rho\sigma_z^{sq}\sigma_z^{mq} - \sigma_z^{sq}\rho\sigma_z^{mq}). \tag{5}$$

In the process of information transmission, the information exchange between control qubits and storage qubits is frequent. Strictly speaking, the information exchange between control qubits and storage qubits should be carried out in the resonant state. However, due to the multi-level structure of actual solid-state qubits and the internal defects of solid material, the hybrid qubits are generally constituted by different types of qubits, all of which will result in internal exchange energy dissipation in the process of information exchange between the control qubits and storage qubits. $L^{(2)}\rho$ exactly describes the exchange energy dissipation of hybrid qubits in the information transmission process, $\Gamma^{sm}$ is the exchange decay rate between control qubits and storage qubits. $L^{(1)}\rho$ reflects the decoherence effect of external environment on the hybrid quantum system. The energy relaxations of control qubits and storage qubits closely relate with their decoherence time $2T_2^{jq} = \left(\Gamma^{jq}\right)^{-1}$ ($j = s, q$).

Since the decoherence time of storage qubits is much longer than that of control qubits, compared to the control qubits, theoretically, $T_2^{mq} \to \infty$. Ref. [2]investigates the asymmetry of the decoherence time of control qubits and storage qubits, and points out that the speed of information exchange between control qubits and storage qubits is very high that each information exchange is equivalent to a quantum measurement. That is to say, there is an anti-Zeno effect between control qubits and storage qubits along with the information exchange. On one hand, it causes the decoherence time of storage qubits to decrease to a finite value, on the other hand, it makes the decoherence time of storage qubits associate with the decoherence time of control qubits. At very low temperature, the reduced decoherence time of storage qubits is, $T_2^{mq} = \lambda T_2^{sq}$. Proportion coefficient $\lambda$ is generally inversely proportional to the coupling strength $g$ between qubits and is proportional to $\Delta = \omega^{sq} - \omega^{mq}$.

## 3. Modulated entanglement evolution

In this section, through studying Lindblad quantum master Eq. (3), we discuss the dynamical evolution of quantum system entanglement.

Suppose that the initial density matrix is only partially coherent, but include an arbitrary degree of nonlocal coherence. This mixed state is easily expressed in the following form,

$$\rho(0) = \frac{1}{3}\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix}, \tag{6}$$

where $a$, $b$, $c$ are independent parameters governing the nature of the initial state of two entangled qubits. Note that the entanglement part of the state depends on the initial phase $z = \sqrt{bc}e^{i\varphi}$ and includes the Bell states as special state. The solution of Eq. (3) under the initial state (6) can be found in Ref. [18] ($\Delta \approx 0$).

We adopt the concurrence entanglement defined by Wootters to measure the system entanglement [19]. For the initial state (6), the concurrence is given by

$$\tilde{C}(t) = \frac{2\sqrt{bc}}{3}f(t)\sqrt{f^2(t)\cos^2\varphi + \left[\cos\left(\frac{\Omega t}{2}\right) - \beta\sin\left(\frac{\Omega t}{2}\right)\right]^2\sin^2\varphi} - \frac{2\sqrt{ad}}{3}, \tag{7}$$

with

$$\Omega = \sqrt{(4g)^2 - \left(\Gamma_2^{sq} + \Gamma_2^{mq} - 2\Gamma^{sm}\right)^2},$$

$$f(t) = \exp\left(-\frac{\Gamma_2^{sq} + \Gamma_2^{mq} - 2\Gamma^{sm}}{2}t\right),$$

$$\beta = \frac{\Gamma_2^{sq} + \Gamma_2^{mq} - 2\Gamma^{sm}}{2\Omega}.$$

From Eq. (7), the entanglement of two qubits relates with the initial quantum state. Only when the coupling qubits are initially in the entangled state, namely $bc > ad$ is satisfied, the entanglement can be maintained in subsequent evolution since $f(t)$ is usually a monotone decreasing function. It also reveals that the decoherence from environment has huge influence on the dynamical evolution of entanglement, which is characterized by the decoherence of each qubit and the exchange decay rate between qubits. It is amazing that when the decoherence time for each qubit is certain, appropriately manipulating the exchange decay rate between qubits benefits the maintenance of quantum entanglement. Inversely, when the exchange decay rate is invariant, controlling the decoherence time of each qubit will help to keep the quantum entanglement. Namely when,

$$\frac{1}{2T_2^{sq}} + \frac{1}{2T_2^{mq}} = \Gamma^{sm}, \tag{8}$$

the dynamical evolution of entanglement between the coupling qubits is independent of the decoherence from environment, and a subspace of non-decoherence is formed. The concurrence only relates with the initial quantum state and the interaction between qubits

$$\tilde{C}(t) = \frac{2\sqrt{bc}}{3}\sqrt{\cos^2\varphi + \cos^2(gt)\sin^2\varphi} - \frac{2\sqrt{ad}}{3}, \tag{9}$$

which indicates that the interaction between qubits will also result in entanglement death and birth.

## 4. Regulation program

It can bee seen from above discussions that the maintenance of quantum entanglement can be realized through manipulating