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Deviation problems in ray optics in the light of the refined unambiguous definitions of angles of incidence, reflection and refraction

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ABSTRACT

This paper makes use of the newly introduced refined unambiguous definitions of angle of incidence, angle of reflection and angle of refraction to give birth to novel vectorial treatment for the calculation of deviation of a ray of light in each of a few cases of ray optics. The problem of calculation of minimum deviation of a ray of light in passing through a prism has also been considered in the light of the refined unambiguous definitions of angle of incidence and angle of refraction. The use of vectorial treatment increases the range of applicability of vector algebra. Furthermore, incorporation of the refined unambiguous definitions of the aforesaid three angles in the novel treatment offered is much clearer leaving no room for confusion and it will enhance and sophisticate the optical field.

1. Introduction

Study of geometrical/ray optics is primarily based on the traditional concepts of angles of incidence, reflection, and refraction [1–10], which have been in regular use for more than two hundred years. All theoretical development of the relevant branch of physics have been made on the basis of such traditional concept and definitions of the aforesaid angles. In 2005, it has been reported by the author in [11] that the long-running concept and definitions of the angles of incidence, reflection, and refraction are ambiguous on account of the fact that such definitions of the said three angles are not in conformity with the fundamental definition of angle in geometry [12], as a result of which the traditional laws of reflection and refraction are also ambiguous, not being able to explain the two typical examples considered in [11]. In order to get rid of the ambiguity, those three angles have been redefined in [11] and such unambiguous definitions of angles of incidence, reflection, and refraction ultimately led to the discovery of the generalized vectorial laws of reflection and refraction [11]. However, the refined definitions of angles of incidence, reflection, and refraction offered in [11] were incomplete in respect of the lower and upper bounds of each of the said three angles. In a recent paper [13], refinement of the definitions of angles of incidence, reflection, refraction, and the critical angle have been made along with specification of the lower and upper bounds of each of the aforesaid angles and a novel discussion of critical angle and total internal reflection has also been presented.

Now that the long-running definitions of angles of incidence, reflection, and refraction stand ambiguous, all theoretical treatment in ray optics which are based on those traditional definitions of the said three angles need immediate replacement in the light of the newly introduced unambiguous definitions of the angles of incidence, reflection, and refraction [13] to enhance and sophisticate the study of optical physics. With that point in mind, the present work is being offered. As a first attempt, the deviation problem in ray optics is considered. Novel vectorial treatment, that takes care of the unambiguous definitions of the aforesaid angles [13], for the calculation of the deviation in each of the cases of reflection/refraction at a plane reflector/surface of discontinuity as well as that on passing through a prism has been offered. Finally, novel derivation of the condition of minimum deviation of a ray of light passing through a prism in the light of the newly introduced unambiguous angles of incidence and refraction [13] has also been presented.

2. Refined unambiguous definitions of angles of incidence, reflection, and refraction

In this section, the refinement made in [13] regarding the definitions of angle of incidence, angle of reflection, and angle of

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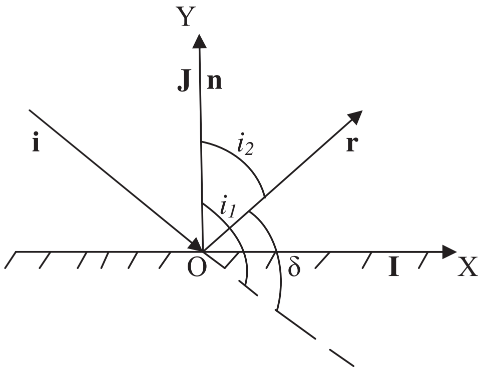

Fig. 1. Diagram showing reflection by a plane mirror.

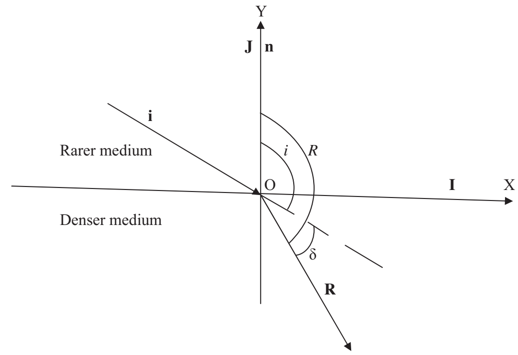

Fig. 2. Diagram showing refraction at a plane surface of discontinuity.

refraction are being presented from the point of view of general interest.

If **n** is a unit vector along the direction of the positive unit normal to the reflector/surface of separation at the point of incidence, **i** is a unit vector along the direction of the incident ray, **r** is a unit vector along the direction of the reflected ray, and **R** is a unit vector along the direction of the refracted ray, then the definitions of unambiguous angles of incidence, reflection, and refraction [13] will be as follows.

Angle of incidence (i): The angle of incidence (i) is the smaller of the angles between the vectors **i** and **n** subject to the condition that $\pi/2 < i \le \pi$, so long as the case considered is a reflection (or a refraction of light as it passes from a rarer to a denser medium). If however it is a case of refraction as light passes from a denser medium to a rarer medium, the angle i must be bounded by the relation $0 \le i < \pi/2$.

Angle of reflection (r): The angle of reflection (r) is the smaller of the angles between the vectors **r** and **n** subject to the condition that $0 \le r < \pi/2$.

Angle of refraction (R): The angle of refraction (R) is the smaller of the angles between the vectors **n** and **R** subject to the condition that, $\pi/2 < R \le \pi$ when the ray of light passes from a rarer medium to a denser medium or $0 \le R < \pi/2$ when the ray of light passes from a denser medium to a rarer medium.

3. Vectorial treatment for the derivation of expression of deviation

In this section, vector algebra has been employed for the derivation of angle of deviation of a ray considering the following three cases.

3.1. Case 1: Deviation of a ray of light due to single reflection by a plane mirror

Vectorial treatment for the calculation of deviation of an incident ray due to reflection by a plane mirror in the light of the refined definitions of unambiguous angle of incidence and angle of reflection has been considered here. For this purpose, let us consider Fig. 1, in which the unambiguous angle of incidence and angle of reflection are represented by i_1 and i_2 respectively.

In this case we have,

$$\mathbf{n} = \mathbf{J}; \quad \mathbf{i} = \sin(\pi - i_1)\mathbf{I} - \cos(\pi - i_1)\mathbf{J} = \sin i_1\,\mathbf{I} + \cos i_1\,\mathbf{J};$$

$$\mathbf{r} = \sin i_2\,\mathbf{I} + \cos i_2\,\mathbf{J}.$$

Then,

$$\mathbf{i}\cdot\mathbf{r} = (\sin i_1\,\mathbf{I} + \cos i_1\,\mathbf{J})\cdot(\sin i_2\,\mathbf{I} + \cos i_2\,\mathbf{J})$$

or,

$$\mathbf{i}\cdot\mathbf{r} = \sin i_1 \quad \sin i_2 + \cos i_1 \quad \cos i_2$$

or,

$$\mathbf{i}.\mathbf{r} = \cos(i_1 - i_2)$$

or,

$$\cos\,\delta = \cos(i_1 - i_2)$$

Hence,

$$\delta = i_1 - i_2$$

Thus in the light of the refined definitions of unambiguous angles of incidence and reflection, the deviation (δ) suffered by a ray of light due to single reflection in a plane mirror is given by, $\delta = i_1 - i_2$.

3.2. Case 2: Deviation of a ray of light due to single refraction at a plane surface of discontinuity

The deviation suffered by a ray of light while passing from one medium to another owing to refraction may be calculated in the light of the refined definitions of unambiguous angle of incidence and angle of refraction with the help of vector algebra. For this purpose, let us consider Fig. 2, in which the unambiguous angle of incidence and angle of refraction are represented by i and R respectively.

Then in this case we must have,

$$\mathbf{n} = \mathbf{J}; \quad \mathbf{i} = \cos\left(i - \frac{\pi}{2}\right)\mathbf{I} - \sin\left(i - \frac{\pi}{2}\right)\mathbf{J} = \sin i\,\mathbf{I} + \cos i\,\mathbf{J};$$

and

$$\mathbf{R} = \sin(\pi - R)\mathbf{I} - \cos(\pi - R)\mathbf{J} = \sin R\,\mathbf{I} + \cos R\,\mathbf{J}$$

Now,

$$\mathbf{i}\cdot\mathbf{R} = (\sin i\,\mathbf{I} + \cos i\,\mathbf{J})\cdot(\sin R\,\mathbf{I} + \cos R\,\mathbf{J})$$

or,

$$\mathbf{i}\cdot\mathbf{R} = \sin i \sin R + \cos i \cos R$$

or,

$$\mathbf{i}\cdot\mathbf{R} = \cos(R - i)$$

or,

$$\cos\,\delta = \cos(R - i)$$

Hence,

$$\delta = R - i$$