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Uniform of four fractional-order nonlinear feedback synchronizations

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ABSTRACT

This paper designs four fractional order nonlinear feedback synchronizations with the simple configuration, followed with their uniform. The closed system's stability is proved based on the fractional order stability theory. Resorted to the fractional order unified chaotic system, it is illustrated that the uniform includes the active, ordinary, dislocated, speed nonlinear feedback synchronizations and their mixed formulations. Numerical simulations show the effectiveness of the proposed methods.

1. Introduction

Fractional calculus dates from 300 years ago, but its applications to physics and engineering are just a recent focus of interest [1–4]. It is found that many systems could be described by the fractional differential equations, such as electromagnetic waves, electrode–electrolyte polarization, electromagnetic wave, dielectric polarization, ultracapacitors and dynamics of fractional order dynamical systems [5–8]. In recently years, fractional order chaotic systems have been investigated and the synchronization of these chaotic systems has attracted more and more researchers' attention. For example Grigorenko et al. proposed the fractional Lorenz system [9]; Hartley et al. proposed the fractional Chua system [10]; Wu et al. found that chaos exists in the fractional order unified system with order less than 3 [11]; Wang and Zhang researched the fractional unified synchronization based on the Pecora and Carroll principle and one-way coupling method [12]; Zhu presented a synchronization method of fractional Chua system via one-way coupling method [13]; Kuntanapreeda designed a robust linear synchronization control of fractional-order unified chaotic systems [14].

On the other hand, due to its simple configuration, easy implementation and easily duplicated from one system to another, the feedback scheme is attractive and has been developed into many forms, such as active synchronization [15], nonlinear feedback synchronization [16], dislocate feedback synchronization [17] and others. But the design of nonlinear feedback synchronization with simple structure and their uniform is still under investigating.

This paper proposed a uniform of the chaotic nonlinear feedback synchronization. The uniform includes the active synchronization, ordinary nonlinear feedback synchronization, dislocated feedback synchronization, speed feedback nonlinear synchronization and their mixed formulations. All the mentioned formulations are illustrated by the fractional order unified chaotic system. Based on the fractional order system stability theory, the closed loop systems' stability is proved. The rest of this paper is organized as follows. In Section 2, the fractional calculus and the fractional system's stability theory is introduced. In Section 3, the uniform of the fractional order unified system's nonlinear synchronization is presented, followed with the simulations to illustrate their effectiveness. Section 4 gives some conclusions.

2. Fractional calculus and fractional system's stability theory

There are many definitions of fractional derivatives [1,2], such as Riemann-Liouville, Grünwald-Letnikov and Caputo definition. The Caputo definition is given as

$$D^{\alpha}x(t) = J^{m-\alpha}x^{(m)}(t), \quad \alpha > 0 \tag{1}$$

where $m = [\alpha]$, i.e., m is the biggest integer which is not less than α. J^{β} is the β-order integral operator:

$$J^{\beta}x(t) = \frac{1}{\Gamma(\beta)}\int_0^t (t-\tau)^{\beta-1} x(\tau)d\tau, \quad \beta > 0 \tag{2}$$

where $\Gamma(\cdot)$ is the Gamma function. As it has traditional physical meanings which can be measured and the fractional derivative of a constant is zero, the Caputo definition is used in this note.

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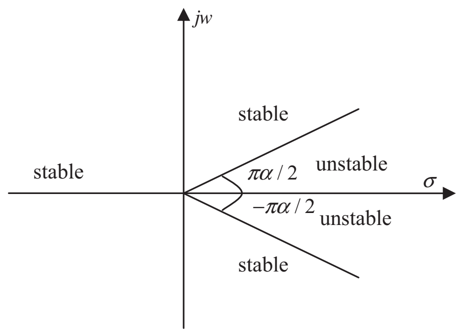

Fig. 1. Stability region of the fractional-order system.

Given the fractional order chaotic system, suppose it can be written as

$$D^{\alpha}X = AX + f(X) \tag{3}$$

where $X \in R^{n\times 1}$, $A \in R^{n\times n}$, $f: R^n \to R^n$, AX is the linear part and $f(X)$ is the nonlinear part.

Let the corresponding response system be

$$D^{\alpha}Y = AY + f(Y) + u(t) \tag{4}$$

where $Y \in R^{n\times 1}$, $u(t)$ is the control function. Define the error of the systems as $e(t) = Y(t) - X(t)$. With Eqs. (3) and (4), the fractional error system can be described as:

$$D^{\alpha}e(t) = Ae(t) + f(Y) - f(X) + u(t) \tag{5}$$

Here, let the control function be

$$u(t) = F(X, Y) + Ke(t) + N \cdot D^{\alpha}e(t) \tag{6}$$

Eq. (6) is called as nonlinear feedback synchronization controller, which is consisting of two parts: The first is nonlinear part $F(X, Y)$, which is used to eliminate the nonlinear effect; the second is linear feedback part $Ke(t) + N\cdot {}_aD_b^{\alpha}e(t)$, and K, N are the designed matrices to assure the closed fractional system's stability. Eq. (6) is regarded as the uniform of some synchronization methods, such as active synchronization, ordinary feedback synchronization, dislocated feedback synchronization and speed feedback synchronization. In the followed section, it is can be verified by the synchronization of fractional unified chaos system.

Theorem 1 ([18]). *For a given autonomous fractional order linear system $D^{\alpha}X = AX$, it is asymptotically stable if and only if $|\arg(\lambda_i(A))| > \alpha\pi/2$, $i = 1, 2, \ldots, n$, where $\arg(\lambda_i(A))$ denotes the argument of the eigenvalue λ_i of A.*

Theorem 1 can be illustrated as Fig. 1.

Theorem 2 ([19]). *Let A be an $n \times n$ symmetric matrix. The following conditions are equivalent:*

(1) *A is negative definite;*
(2) *All the eigenvalues of A are negative;*
(3) *The determinants, $(-1)^k\det(A_k) > 0$ for $1 \le k \le n$, i.e., $\det(A_1) < 0$, $\det(A_2) > 0, \ldots, (-1)^n\det(A_n) = (-1)^n\det(A) > 0$.*

3. Fractional order unified system's nonlinear feedback synchronizations

It is the same as the classical unified chaotic system, the fractional-order unified system could be regarded as the system that bridges the gap among the fractional-order Lorenz system, the fractional-order Lü system, and the fractional-order Chen system. It is described by [20,21]

$$\begin{cases} D^{\alpha_1}x = (25a + 10)(y - x) \\ D^{\alpha_2}y = (28 - 35a)x + (29a - 1)y - xz \\ D^{\alpha_3}z = \dfrac{xy - (a + 8)z}{3} \end{cases} \tag{7}$$

where $a \in [0, 1]$ is the parameter of the system and $0 < \alpha_i < 1$, $i = 1, 2, 3$ is the fractional order. Some examples of the chaotic attractor of the system (7) with $\alpha_1 = 0.985$, $\alpha_2 = 0.99$, $\alpha_3 = 0.99$, $a = 0.2$ and $a = 0.9$ are shown in Figs. 2 and 3, respectively.

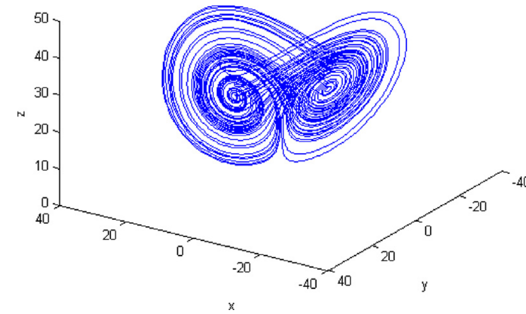

Fig. 2. The fractional unified system with $a = 0.2$, $\alpha_1 = 0.985$, $\alpha_2 = 0.99$, $\alpha_3 = 0.99$.

Suppose the master system and the slave system can be expressed as

$$\begin{cases} D^{\alpha_1}x_m = (25a + 10)(y_m - x_m) \\ D^{\alpha_2}y_m = (28 - 35a)x_m + (29a - 1)y_m - x_m z_m \\ D^{\alpha_3}z_m = \dfrac{x_m y_m - (a + 8)z_m}{3} \end{cases} \tag{8}$$

and

$$\begin{cases} D^{\alpha_1}x_s = (25a + 10)(y_s - x_s) + u_1 \\ D^{\alpha_2}y_s = (28 - 35a)x_s + (29a - 1)y_s - x_s z_s + u_2 \\ D^{\alpha_3}z_s = \dfrac{x_s y_s - (a + 8)z_s}{3} + u_3 \end{cases} \tag{9}$$

where the subscribe m and s stand for the master and slave, respectively.

Define the error variables as $e_x = x_s - x_m$, $e_y = y_s - y_m$, $e_z = z_s - z_m$, and then the error system can be obtained as

$$\begin{cases} D^{\alpha_1}e_x = (25a + 10)(e_y - e_x) + u_1 \\ D^{\alpha_2}e_y = (28 - 35a)e_x + (29a - 1)e_y - z_m e_x - x_s e_z + u_2 \\ D^{\alpha_3}e_z = \dfrac{y_m e_x + x_s e_y - (a + 8)e_z}{3} + u_3 \end{cases} \tag{10}$$

The controller is selected as the followed form

$$u(t) = [u_1, u_2, u_3]^T = F(X, Y) + Ke(t) + N \cdot D^{\alpha}e(t) \tag{11}$$

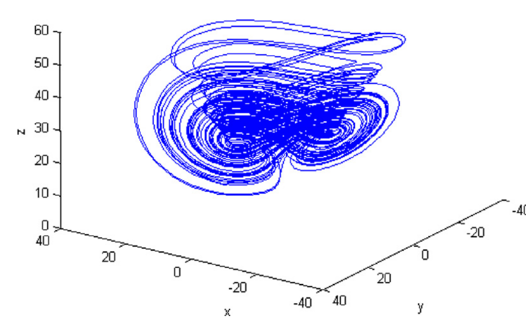

Fig. 3. The fractional unified system with $a = 0.9$, $\alpha_1 = 0.985$, $\alpha_2 = 0.99$, $\alpha_3 = 0.99$.