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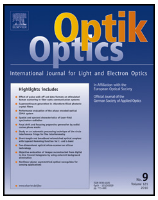

Achromatic half-wave combination of birefringent plates

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ABSTRACT

Half-wave retarders are primarily used in rotating the plane of polarization of polarized light. These retarders usually exhibit strong wavelength dependence. In this paper, the design and characteristics of an achromatic half-wave plate, formed by a cascaded system of birefringent plates, have been studied. Pancharatnam proposed a combination of three retarders in series and discussed the possibility of fabricating reasonably good achromatic quarter-wave plate with a suitable combination of their retardance. The system studied here is similar in construction to Pancharatnam configuration and behaves as an achromatic half-wave plate over a wide spectral range. The proposed configuration exhibits a maximum variation of only about ±1.4 degree over the entire wavelength range of 500–750 nm. In our analysis we have used Jones matrix formalism for the derivation of the general expression for the equivalent retardation and the azimuth of the combination.

1. Introduction

A half-wave plate is a useful polarization device and is required in many experimental layouts involving polarization optics. This device is ubiquitously used for rotating the plane of vibration or polarization ellipse by any pre-specified amount. For example, a $\lambda/2$-plate can convert an oblique ellipse to a symmetrical ellipse by adjusting the azimuth of the half-wave plate. We know that the retardance of a single plate birefringent retarder is highly chromatic, as is obvious from the expression for the retardation δ of a birefringent plate between the vibrations along two privileged directions of the plate.

$$\delta = \frac{2\pi}{\lambda_0}\Delta n d \qquad (1)$$

where Δn is the birefringence of material of the plate. Since Δn of birefringent materials commonly used for making a retarder does not appreciably vary within the visible range, it is evident that the retardation varies almost inversely with the wavelength of light. If an experimental layout using polarized polychromatic light demands a use of a $\lambda/2$-plate, it must act as a $\lambda/2$-plate for the entire range of wavelengths constituting the polychromatic light used. Thus, an achromatic half-wave plate would be a useful polarizing device. Design of achromatic retarders, over at least the wavelength range of interest, therefore assumes significance. Different techniques have been used to attain such achromatism using both passive devices as well as active devices. In some of the cases the same birefringent material has been used to fabricate the retarder combination, whereas in other cases different materials are employed. Retarders with the best achromatic characteristics are based on the principle of unequal phase retardation between the fast and slow components of an incident beam on total internal reflection at the glass-air interface. An example of this is the Fresnel rhomb. This subject has been covered in detail by Bennett [1]. Several researchers worked in this field to design various types of achromatic retarders using this principle of total internal reflection in Fresnel rhomb [2–8]. However, in general, retarders based on the principle of the Fresnel rhomb are limited in aperture due to their geometry. Recently achromatic retarders are designed using liquid crystal cells [9–12]. Achromatic phase retarders have also been designed using sub wavelength grating structures [13–16] known as *form birefringence*. Composite plates, consisting of two or more plates of the same material whose axes are oriented at the appropriate angles, can also be used as achromatic circular polarizers or achromatic polarization rotators [17]. In fact Pancharatnam proposed one of the most successful combinations of three plates which were produced of the same material [18], with the extreme retardation plates having similar optical axes orientations and retardations while the central plate has a different retardation and orientation. Subsequently, achromatic quarter-wave and half-wave retarders have been produced using a combination of two or three plates of different

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birefringent materials [19–21] to minimize retardation dispersion across a broad wavelength range. Hariharan has devised a procedure to optimize the choice of materials for broadband retarders [22]. Hariharan and Malacara proposed a combination of two quarter-wave plates and one half-wave plate of the same material, which can be used as an achromatic half-wave retarder [23]. Hariharan has also showed that two plates of different birefringent materials, when suitably configured, exhibits achromatic behavior over a limited range of wavelengths [24]. A reconfigurable achromatic half-wave and quarter-wave retarder in near infrared has been proposed using three crystalline quartz plates [25]. A cascaded system of four plates has also been studied [26]. However, one of the early works in this domain was done by Destriau et al. [27] where the authors showed that an almost achromatic quarter wave retarder could be obtained by juxtaposing two retarders – one half-wave and the other quarter-wave plate. However, they used the Poincaré sphere method to design the system and the variation of azimuth is not shown in their work. Two different configurations for producing achromatic quarter-wave retardation using only two plates have also been studied recently where the problems of the works of Destriau has been removed [28]. However, in most of the cases, the retarders designed are either costly or show the performance over only a very limited range.

2. Formulation of the problem

The combination considered by us is a cascaded system of three birefringent plates, as shown in Fig. 1. For general analysis of such a system of birefringent plate, we assume δ_1 to be the retardation of the two extreme plates and δ_2 to be that of the central plate. The angle ϕ is the inclination between the fast axis of the central plate and the fast axis of the two extreme plates.

We now calculate the retardation of the achromatic combination using the well-known Jones matrix method. The Jones matrix of a retarder whose fast axis coincide with the x-axis is given by

$$C(\delta, 0) = \begin{vmatrix} e^{i\delta/2} & 0 \\ 0 & e^{-i\delta/2} \end{vmatrix} \qquad (2)$$

where δ is the phase difference introduced by the retarder.

Again, the characteristic Jones matrix of an oblique retarder whose fast axis makes an angle ϕ with the x-axis is

$$C(\delta, \phi) = \begin{vmatrix} \cos\frac{\delta}{2} + i\sin\frac{\delta}{2}\cos 2\phi & i\sin\frac{\delta}{2}\sin 2\phi \\ i\sin\frac{\delta}{2}\sin 2\phi & \cos\frac{\delta}{2} - i\sin\frac{\delta}{2}\cos 2\phi \end{vmatrix} \qquad (3)$$

To make the system more generalized we consider the retardation value of the central plate as δ_2, which is different from the two extreme plates. Hence, the Jones matrix of the cascaded system considered is

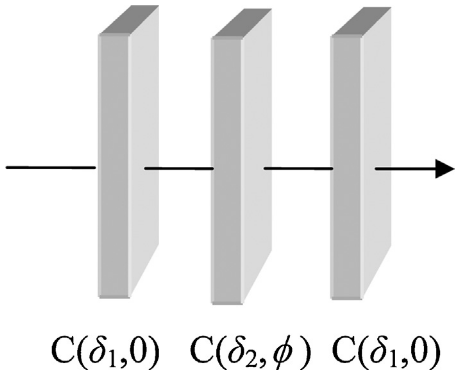

Fig. 1. Configuration of the three birefringent plates.

$$C(\Delta, \psi) = C(\delta_1, 0)C(\delta_2, \phi)C(\delta_1, 0) \qquad (4)$$

where Δ is the equivalent retardation of the combination and Ψ is the azimuth of the fast axis of the combination.

Replacing δ by Δ and ϕ by Ψ in Eq. (3), we get

$$C(\Delta, \Psi) = \begin{vmatrix} \cos\frac{\Delta}{2} + i\sin\frac{\Delta}{2}\cos 2\Psi & i\sin\frac{\Delta}{2}\sin 2\Psi \\ i\sin\frac{\Delta}{2}\sin 2\Psi & \cos\frac{\Delta}{2} - i\sin\frac{\Delta}{2}\cos 2\Psi \end{vmatrix} \qquad (5)$$

Writing the Jones matrices of the individual elements of the combination explicitly in the right hand side of Eq. (4) we get

$$\begin{aligned} C(\Delta, \psi) &= \begin{vmatrix} e^{i\delta_1/2} & 0 \\ 0 & e^{-i\delta_1/2} \end{vmatrix} \begin{vmatrix} \cos\frac{\delta_2}{2} + i\sin\frac{\delta_2}{2}\cos 2\phi & i\sin\frac{\delta_2}{2}\sin 2\phi \\ i\sin\frac{\delta_2}{2}\sin 2\phi & \cos\frac{\delta_2}{2} - i\sin\frac{\delta_2}{2}\cos 2\phi \end{vmatrix} \\ &\times \begin{vmatrix} e^{i\delta_1/2} & 0 \\ 0 & e^{-i\delta_1/2} \end{vmatrix} \\ &= \begin{vmatrix} e^{i\delta_1}\cos\frac{\delta_2}{2} + ie^{i\delta_1}\sin\frac{\delta_2}{2}\cos 2\phi & i\sin\frac{\delta_2}{2}\sin 2\phi \\ i\sin\frac{\delta_2}{2}\sin 2\phi & e^{-i\delta_1}\cos\frac{\delta_2}{2} - ie^{-i\delta_1}\sin\frac{\delta_2}{2}\cos 2\phi \end{vmatrix} \end{aligned} \qquad (6)$$

Equating the real and imaginary parts of Eqs. (5) and (6), we obtain

$$\cos\frac{\Delta}{2} = \cos\delta_1\cos\frac{\delta_2}{2} - \sin\delta_1\sin\frac{\delta_2}{2}\cos 2\phi \qquad (7)$$

and

$$\cot 2\Psi = \operatorname{cosec} 2\phi\left(\sin\delta_1\cot\frac{\delta_2}{2} + \cos\delta_1\cos 2\phi\right) \qquad (8)$$

Eqs. (7) and (8) are the governing equations, where Δ and Ψ are the resultant retardance and azimuth of the light transmitted through the proposed arrangement.

3. The design procedure

The aim of this study is to achieve an achromatic half-wave plate over the wavelength range 500–750 nm. To achieve this, a combination of three half-wave plates, all designed for 600 nm, have been considered. We have calculated the retardation of the overall system for the design wavelength (i.e., 600 nm), the two extreme wavelengths (i.e., 500 nm and 750 nm), and two other intermediate wavelengths (i.e., 550 nm and 700 nm) for different orientations of the central plate. The retardation for the combination is plotted in Fig. 2. As seen from Fig. 2, all the five graphs coincide at 58.27° and 121.7°. This shows that the combination will behave as an achromatic retarder at either of these two orientations of the central plate. Moreover, it is also seen that the difference in retardation for the two extreme wavelengths is maximum when the orientation of the central plate is either 0° or 180°. This indicates that the retardation for the combination will vary maximum at this orientation.

The retardation δ introduced by a birefringent plate between the two orthogonal components of light is given by

$$\delta = \frac{2\pi}{\lambda_0}\Delta n d \qquad (9)$$

where Δn is the birefringence of the material of the retarders, d is the thickness of each plate and λ_0 is the design wavelength. For numerical analysis of the chromatic behavior of the cascaded system of birefringent plates considered, we have assumed that each