Optik 126 (2015) 361–367

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Orthogonal multilinear discriminant analysis and its subblock tensor analysis version

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ARTICLE INFO

Article history:
Received 7 December 2013
Accepted 20 August 2014

Keywords:
Orthogonal multilinear discriminant analysis (OMDA)
Subblock tensor analysis
Subblock orthogonal multilinear discriminant analysis (SOMDA)
Gait recognition

ABSTRACT

This paper introduces an orthogonal multilinear discriminant analysis (OMDA) algorithm for gait recognition. The discriminant feature vectors of OMDA are orthogonal to each other. With the advantage of extracting a portion of local information and reducing computational complexity, the subblock tensor analysis is employed to OMDA, named subblock orthogonal multilinear discriminant analysis (SOMDA). Considering that the vectors from different subblocks have different contributions to recognition, these vectors are given different weights and synthesized into a whole vector in the recognition process. We have conducted a comparative study on gait recognition to evaluate OMDA and SOMDA in terms of classification. With the tensor vectorization methods according to both variance and class discriminability, the OMDA-based recognition algorithm indicates that it outperforms other multilinear subspace solutions such as MPCA, MPCA + LDA, GTDA, DATER and UMDA. In the subblock experiments, it indicates that SOMDA is an improvement over OMDA.

1. Introduction

In recent years, the high demand for intelligent system applications of artificial intelligence techniques are increasing due to the development of information technology. To date, a number of techniques have been provided and studied for authentication and identification, most using smart cards, face [1], fusion information obtained from face, teeth and voice modalities [2] and so on. They not only can withstand against password attacks, but can yield acceptable performance. However, the above-mentioned information is not available at a distance. Gait has the advantages of being noninvasive, non-contact, insensitive to environment and hard to conceal, and it is probably the only biometric available at a distance [3].

Lu et al. [4] proposed Multilinear Principal Component Analysis (MPCA) and operated directly on the tensorial representations rather than vectorized versions. It is well understood that such reshaping (vectorization) methods as Principal Component Analysis (PCA), Independent Components Analysis (ICA), Linear Discriminant Analysis (LDA) break the natural structure and correlation in the original data, reduce redundancies and higher order dependencies presentations in the original data set and lose potentially more compact or useful representations that can be obtained in the original tensorial forms. A serial of novel approaches to dimensionality reduction of multidimensional data, where the input data are represented as their natural multidimensional tensors, are emerging. They can be sorted into two classes, namely tensor-to-tensor projection (TTP) and tensor-to-vector projection (TVP). The typical methods in TTP include General Tensor Discriminant Analysis (GTDA) [5] and Discriminant Analysis with Tensor Representation (DATER) [6]. The criterion of DATER is to maximize the tensor-based scatter ratio with a disadvantage of non-convergence in an iterative solution while the criterion of GTDA is to maximize the scatter difference with a disadvantage of having difficulty in difference parameter selection. The typical method in TVP is tensor rank-one discriminant analysis (TR1DA) algorithm [7] which obtains a number of rank-one projections with the scatter difference criterion from the repeatedly calculated residues of the original tensor data. However, TR1DA does not take the correlations among features into account. Lu et al. [8] proposed Uncorrelated Multilinear Discriminant Analysis (UMDA) to extract

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http://dx.doi.org/10.1016/j.ijleo.2014.08.127
0030-4026/

uncorrelated features directly from tensor data through solving a TVP. The UMDA-based recognition algorithm is validated to outperform MPCA, DATER, GTDA and TR1DA on the gait recognition task. Multilinear extensions of linear graph-embedding algorithms were also introduced [9,10] in a similar way as the existing multilinear subspace learning (MSL) algorithms reviewed in this paper.

In comparison with the state-of-the-art, the contributions of this paper are:

1) The orthogonal multilinear discriminant analysis (OMDA) algorithm for orthogonal discriminant feature extraction from tensors is introduced. As a multilinear extension of LDA, this algorithm not only obtains discriminative features by maximizing the scatter ratio, but also enforces a constraint so that the derived features are orthogonal. This algorithm differs from the classical approaches which are based on vectors and can destroy the natural structure and correlation in the original data. OMDA can overcome the under-sampling problems.
2) Subblock orthogonal multilinear discriminant analysis (SOMDA) proposed by us is a subblock tensor analysis version of OMDA. SOMDA with the solution of subblocking the tensor into small portions can extract more detailed information and reduce computational complexity.

The remainder of this paper is organized as follows. Section 2 presents orthogonal multilinear discriminant analysis (OMDA) algorithm and summarizes its computational complexity. The two proposals of subblock orthogonal multilinear discriminant analysis (SOMDA) are described in Section 3. Performance of OMDA and its subblock tensor, namely SOMDA for gait recognition are presented in Section 4. This paper ends with a conclusion in Section 5.

2. Orthogonal multilinear discriminant analysis

As the basic multilinear operation of orthogonal multilinear discriminant analysis (OMDA), the algorithm of multilinear principal component analysis (MPCA) is reviewed, and then OMDA and its computational complexity analysis are described in detail.

2.1. Multilinear principal component analysis

MPCA is a multilinear algorithm; it performs dimensionality reduction in all tensor modes and seeks those bases in each mode that allow projected tensors to capture most of the variation in the original tensors.

Let a tensor samples set $\{x_m \in R^{I_1 \times I_2 \times \cdots \times I_N}, m = 1, \ldots, M\}$ be a set of M training samples in a tensor space $R^{I_1} \otimes R^{I_2} \otimes \cdots \otimes R^{I_N}$, where $I_n (n = 1, \ldots, N)$ denotes the dimension of n-mode for tensor objects. Multilinear transformation matrices $\{\tilde{U}^{(n)} \in R^{I_n \times P_n}, n = 1, \ldots, N\}$ are defined in order to map the original tensor space $R^{I_1} \otimes R^{I_2} \otimes \cdots \otimes R^{I_N}$ into a new tensor space $R^{P_1} \otimes R^{P_2} \otimes \cdots \otimes R^{P_N} (P_n < I_n, n = 1, \ldots, N)$:

$$y_m = x_m \times_1 \tilde{U}^{(1)\mathrm{T}} \times_2 \tilde{U}^{(2)\mathrm{T}} \times_3 \ldots \times_N \tilde{U}^{(N)\mathrm{T}}, \tag{1}$$

where $P_n (n = 1, \ldots, N)$ denotes the dimension for each mode.

$y_m \in R^{P_1} \otimes R^{P_2} \otimes \cdots \otimes R^{P_N}, m = 1, \ldots, M$ can capture the directions of largest variation from the original tensor data, which is measured by the total scatter Ψ_y

$$\{\tilde{U}^{(n)}, n = 1, \ldots, N\} = \underset{\tilde{U}^{(n)}, n=1,\cdots,N}{\arg\max}\ \Psi_y \tag{2}$$

There is no close solution to Eq. (2) when it is optimized for $\{\tilde{U}^{(n)}, n = 1, \ldots, N\}$ at the same time. This problem can be solved as a problem of N projections to N subspaces; therefore it needs an iterative solution. Projection matrices $\{\tilde{U}^{(i)}, i = 1, \ldots, n-1, n+1, \ldots, N\}$ are fixed when $\tilde{U}^{(n)}$ is calculated. $\tilde{U}^{(n)}$ is obtained by an eigenvalue decomposition of $\Phi^{(n)}$ and choosing the eigenvectors corresponding to the P_n largest eigenvalues.

$$\Phi^{(n)} = \sum_{m=1}^{M} (X_{m(n)} - \bar{X}_{(n)}) \tilde{U}_{\Phi^{(n)}} \tilde{U}^{\mathrm{T}}_{\Phi^{(n)}} (X_{m(n)} - \bar{X}_{(n)})^{\mathrm{T}}, \tag{3}$$

$$\tilde{U}_{\Phi^{(n)}} = \tilde{U}^{(n+1)} \otimes \tilde{U}^{(n+2)} \otimes \cdots \otimes \tilde{U}^{(N)} \otimes \tilde{U}^{(1)} \otimes \tilde{U}^{(2)} \otimes \cdots \otimes \tilde{U}^{(n-1)}, \tag{4}$$

where $X_{m(n)}$ denotes n-mode matrix of the m-th sample, and $\bar{X}_{(n)}$ denotes the n-mode mean matrix of these M training samples.

$$\bar{X}_{(n)} = \frac{1}{M} \sum_{m=1}^{M} X_{m(n)}. \tag{5}$$

$\{P_n, n = 1, \ldots, N\}$ is assumed to be known or determined by maximizing the following criterion

$$\{\tilde{U}^{(n)}, P_n, n = 1, \ldots, N\} = \underset{\tilde{U}^{(n)}, n=1,\cdots,N; P_1,\cdots,P_N}{\arg\max} \Psi_y, \tag{6}$$

subject to

$$\frac{\Pi_{n=1}^{N} P_n}{\Pi_{n=1}^{N} I_n} < \Omega, \tag{7}$$

where Ω, whose value is usually determined by users, is the rate between the targeted dimensionality and the original tensor dimensionality. We can now consider N mappings to reduced dimensions $P_n (n = 1, \ldots, N)$ by specifying that the new components must account for at least a fraction Ω of the total dimension $\Pi_{n=1}^{N} I_n$. It also can be determined by the rate $testQ^{(n)}$ between the total scatter in the n-mode after the truncation of the n-mode eigenvectors exceeding the P_n-th and the total scatter without truncation.

$$testQ^{(n)} = \frac{\sum_{i_n=1}^{P_n} \lambda_{i_n}^{(n)*}}{\sum_{i_n=1}^{I_n} \lambda_{i_n}^{(n)*}}, \tag{8}$$

where $\lambda_{i_n}^{(n)*}$ is the i_n-th full-projection n-mode eigenvalue. $\sum_{i_n=1}^{I_n} \lambda_{i_n}^{(n)*} = \Psi_x$ is the total scatter of original samples.

2.2. Orthogonal multilinear discriminant analysis (OMDA)

Eq. (1) compresses each tensor sample of size $\Pi_{n=1}^{N} I_n$ into a new tensor with the size of $\Pi_{n=1}^{N} P_n$. But the new tensor has redundancy which needs to be reduced by the tensor vectorization and selection. There are two ways of tensor vectorization with feature vector sorting. One is according to variance, namely selecting fewer vectors according to $\lambda_{i_n}^{(n)*}$ in descending order; the other is according to class discriminability $\Gamma_{p_1 p_2 \ldots p_N}$:

$$\Gamma_{p_1 p_2 \ldots p_N} = \frac{\sum_{c=1}^{C} N_c [\bar{y}_c(p_1, p_2, \ldots, p_N) - \bar{y}(p_1, p_2, \ldots, p_N)]^2}{\sum_{m=1}^{M} [y_m(p_1, p_2, \ldots, p_N) - \bar{y}_{c_m}(p_1, p_2, \ldots, p_N)]^2}, \tag{9}$$

where M and C are the numbers of samples and classes respectively. $N_c (c = 1, \ldots, C)$ is the number of samples from class c. $\bar{y}_{c_m}$ is the class mean of the m-th sample belonging to class C_m in the projected tensor subspace. $\bar{y}_c$ is the mean of class c in the projected tensor subspace. y is the mean of all tensors projected.

After all the training samples $y_m (m = 1, \ldots, M)$ are transformed into vectors $x_m (m = 1, \ldots, M)$, they are arranged in a new matrix $\aleph \in R^{p \times M}$, whose column x_m corresponds to a vector sample, where p $(p = \Pi_{n=1}^{N} P_n)$ is the dimension of feature. Considering a linear