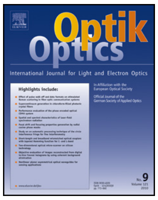
Optik 125 (2014) 2374–2377

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo



Nonlinear intersubband optical absorption in semiparabolic quantum wells

Guang-hui Wang*

Laboratory of Nanophotonic Functional Materials and Devices, South China Normal University, Guangzhou 510006, People's Republic of China

ARTICLE INFO

Article history:
Received 22 May 2013
Accepted 16 October 2013

Keywords:
Nonlinear optical absorption
Semiparabolic quantum well
Density matrix approach

PACS:
42.65. −k
71.35.Cc
78.67.De

ABSTRACT

The linear and third-order nonlinear optical absorptions in semiparabolic quantum wells are studied in detail. Analytic formulas for the linear and third-order nonlinear optical absorption coefficients are obtained using the compact density matrix approach. Based on this model, numerical results are presented for typical *GaAs/AlGaAs* semiparabolic quantum wells. The results show that the factors of the incident optical intensity and the semiparabolic confinement frequency have great influences on the total optical absorption coefficients.

1. Introduction

In the past few years, the nonlinear optical properties of semiconductor quantum well and quantum dot nanostructures, in particular the second- and third-order optical nonlinearities have attracted much attention in the theoretical and applied physics sides [1–7], because of their novel physical properties and promise for potential applications. Furthermore, quantum confinement of carriers in these low-dimensional semiconductor nanostrucures lead to the formation of discrete energy levels and the drastic change of optical absorption spectra [8]. One of the most remarkable properties of these low-dimensional electronic systems is that the optical transitions between the size-quantized subbands are feasible. Recently, the linear intersubband optical absorption within the conduction band of a *GaAs* quantum well has been studied experimentally without an electric field [9], and with an electric field [10]. The fact that a very large dipole strength and a narrow band width were observed suggests that the intersubband optical transitions in a quantum well may have very large optical nonlinearities. Nonlinear intersubband optical absorption in a semiconductor quantum well was calculated by Ahn et al. [11]. Intersubband optical absorption in coupled quantum wells under an applied electric field was studied by Yuh and Wang [12]. In 1993,Cui et al. [13] studied absorption saturation of intersubband optical transitions in $GaAs/Al_xGa_{1-x}As$ multiple quantum wells in experiment, and they tested the saturation optical intensity for $I_s = 0.67\,\mathrm{MW/cm^2}$. In 2003, refractive index changes induced by the incident optical intensity in semiparabolic quantum wells were investigated by the present author [14]. In 2010,Zhang et al. [15] studied nonlinear optical absorption coefficients and refractive index changes in a two-dimensional system. Optical absorption and refractive index changes in a two-dimensional quantum ring with an applied magnetic field were investigated by Xie [16]. From fundamental and practical points of view, These linear and nonlinear size-quantized transitions have the potential for device applications in far-infrared laser amplifiers, photodetectors, and high-speed electrooptical modulators [17,18].

In this paper, the linear and third-order nonlinear intersubband optical absorptions in semiparabolic quantum wells are studied in detail. In Section 2, analytical formulas for the linear, third-order nonlinear and total optical absorption coefficients are derived by using the compact-density-matrix method and an iterative procedure. Numerical results and discussion are presented in Section 3. We find that the incident optical intensity and the semiparabolic confinement frequency have great influence on the total optical absorption coefficients.

---

* Tel.: +86 2039310083; fax: +86 2039310083.
E-mail address: wanggh@scnu.edu.cn

0030-4026/$ – see front matter © 2014 Published by Elsevier GmbH.
http://dx.doi.org/10.1016/j.ijleo.2013.10.116

2. Theory

The effective-mass Hamiltonian for an electron in a semiparabolic quantum well can be written as

$$H = -\frac{\hbar^2}{2m^*}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]+V(z) \tag{1}$$

where z represents the growth direction, $\hbar$ is Planck's constant, and m^* is the conduction-band effective mass, which will be taken to be constant in the rest of the paper. $V(z)$ is the confining potential, namely

$$V(z) = \begin{cases} \frac{1}{2}m^*\omega_0^2 z^2, & z>0 \\ \infty, & z<0 \end{cases} \tag{2}$$

where ω_0 is the semiparabolic confinement frequency (see Fig. 1).

The eigenfunctions $\psi_{n,k}(r)$ and eigenenergies $\varepsilon_{n,k}$ are solutions of the Schrödinger equation $H\psi_{n,k}(r)=\varepsilon_{n,k}\psi_{n,k}(r)$ and are given by

$$\psi_{n,k}(r) = A^{-1/2}\varphi_n(z)U_c(r)e^{ik_\parallel . r_\parallel}, \tag{3}$$

and

$$\varepsilon_{n,k} = E_n + \frac{\hbar^2}{2m^*}|k_\parallel|^2, \tag{4}$$

where A is the area of the well, $k_{||}$ and $r_{||}$ are the wave vector and coordinates in the xy plane, and $U_c(r)$ is the periodic part of the Bloch function in the conduction band at $k=0$. φ_n and E_n are the envelope wave function and the transverse energy of the nth subband, solutions of one-dimensional Schrödinger equation $H_0\varphi_n(z)=E_n\varphi_n(z)$, where H_0 is the z part of the Hamiltonian H in Eq. (1), *i.e.*, $H_0=-(\hbar^2/2m^*)(d^2/dz^2)+V(z)$, which eigenfunction and eigenenergy are given by

$$E_n = \left(2n+\tfrac{3}{2}\right)\hbar\omega_0 \quad (n=0,1,2,\ldots,) \tag{5}$$

$$\varphi_n(z) = N_n \exp\left[-\tfrac{1}{2}\alpha^2 z^2\right] H_{2n+1}(\alpha z), \quad (z>0), \tag{6}$$

where $\alpha=\sqrt{m^*\omega_0/\hbar}$, $N_n=\left[\alpha^{-1}\sqrt{\pi}2^{2n}(2n+1)!\right]^{-\frac{1}{2}}$ is the normalization constant, and $H_{2n+1}(\alpha z)$ are the Hermite polynomials.

Therefore the dipolar transitional matrix element can be written as [1]

$$\begin{aligned}\left\langle\psi_n|q|z|\psi_m\right\rangle &= A^{-1}\int e^{-i(k_\parallel^n-k_\parallel^m).r_\parallel}dr_\parallel\left\langle\varphi_n|q|z|\varphi_m\right\rangle \\ &= \delta_{k_\parallel^n,k_\parallel^m}\left\langle\varphi_n|q|z|\varphi_m\right\rangle,\end{aligned} \tag{7}$$

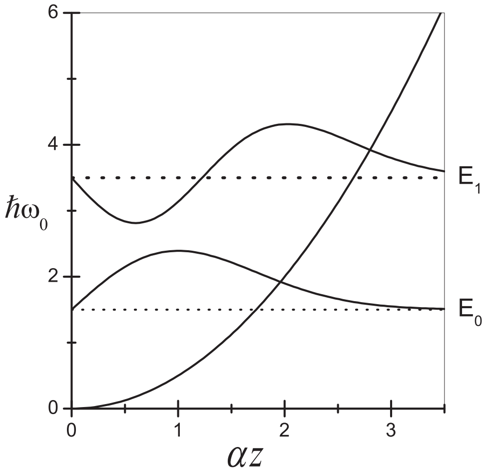


Fig. 1. Schematic diagrams for energy levels and wave functions of a semiparabolic quantum well.

where δ is the Kronecker delta function, and q is the electronic charge.

Next we will derive the expression of the linear and third-order nonlinear optical absorption coefficients in the model. Let us consider an electromagnetic field with frequency ω, which is incident with a polarization vector normal to the quantum well. The system is excited by an electromagnetic field

$$\begin{aligned}E(t) &= E_0\cos(\omega t) \\ &= \tilde{E}e^{i\omega t}+\tilde{E}e^{-i\omega t}.\end{aligned} \tag{8}$$

Let the sign ρ denote the one-electron density matrix for this regime. Then the evolution of the density matrix ρ obeys the following time-dependent Schrödinger equation

$$\frac{\partial\rho_{ij}}{\partial t} = (i\hbar)^{-1}[H_0-qzE(t),\rho]_{ij}-\Gamma_{ij}(\rho-\rho^{(0)})_{ij,} \tag{9}$$

where $\rho^{(0)}$ is the unperturbed density matrix, and Γ_{ij} is the relaxation rate. For simplicity, we will assume in the following only two different Γ_{ij} values: $\Gamma_1=1/T_1$ for $i=j$ is the diagonal relaxation rate, and $\Gamma_2=1/T_2$ for $i\neq j$ is the off-diagonal relaxation rate. T_1 is a population relaxation time, which can be enhanced by storing the excited electrons on a metastable level. T_2 is certainly governed by intrinsic mechanisms such as electron–electron interaction or optical-phonon emission for an excitation energy. Eq. (9) is solved by using the usual iterative method [1], then

$$\rho(t) = \sum_n \rho^{(n)}(t), \tag{10}$$

with

$$\begin{aligned}\frac{\partial\rho_{ij}^{(n+1)}}{\partial t} &= \frac{1}{i\hbar}\left\{[H_0,\rho^{(n+1)}]_{ij}-i\hbar\Gamma_{ij}\rho_{ij}^{(n+1)}\right\} \\ &\quad -\frac{1}{i\hbar}[qz,\rho^{(n)}]_{ij}E(t).\end{aligned} \tag{11}$$

The electronic polarization can be expanded as

$$\begin{aligned}P(t) &= \varepsilon_0\chi^{(1)}\tilde{E}e^{i\omega t}+\varepsilon_0\chi^{(2)}_{2\omega}\tilde{E}^2e^{2i\omega t}+\varepsilon_0\chi^{(2)}_0\tilde{E}^2+\varepsilon_0\chi^{(3)}\tilde{E}^3e^{i\omega t} \\ &\quad +\varepsilon_0\chi^{(3)}_{3\omega}\tilde{E}^3e^{3i\omega t}+\cdots+\text{c.c.}\end{aligned} \tag{12}$$

The dc optical rectification term $\varepsilon_0\chi_0^{(2)}\tilde{E}^2$ will be neglected due to the small size of $\chi_0^{(2)}$. Contributions due to higher order harmonic terms in ω will also be neglected. These assumptions yield the approximate form for $P(t)$ as

$$P(t) \approx \varepsilon_0\chi^{(1)}\tilde{E}e^{i\omega t}+\varepsilon_0\chi^{(3)}\tilde{E}^3e^{i\omega t}+c.c.. \tag{13}$$

For simplicity, we shall confine our attention to two-level systems only for electronic transitions. Hereafter, the ground state will be denoted by g, the first excited state by e, respectively.

The analytical forms of the linear $\chi^{(1)}$ and the nonlinear $\chi^{(3)}$ susceptibilities are given as follows by the same procedure as Ref. [6]. First, for the linear term

$$\varepsilon_0\chi^{(1)}(\omega) = \frac{N|M_{eg}|^2}{\hbar\omega_{eg}-\hbar\omega-i\hbar\Gamma_{ge}}. \tag{14}$$

For the third-order term

$$\begin{aligned}\varepsilon_0\chi^{(3)}(\omega) &= \frac{-N|M_{eg}|^2|E|^2}{\hbar\omega_{eg}-\hbar\omega-i\hbar\Gamma_{ge}} \\ &\left[\frac{4|M_{eg}|^2}{(\hbar\omega_{eg}-\hbar\omega)^2+(\hbar\Gamma_{ge})^2}-\frac{(M_{ee}-M_{gg})^2}{(\hbar\omega_{eg}-i\hbar\Gamma_{ge})(\hbar\omega_{eg}-\hbar\omega-i\hbar\Gamma_{ge})}\right]\end{aligned} \tag{15}$$