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Spectral characteristic of a pulsed hollow Gaussian beam passing through a circular aperture

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ABSTRACT

The spectral characteristics of a pulsed hollow Gaussian beam passing through a circular aperture are studied. Based on the vectorial Rayleigh diffraction integrals, the analytical expressions of the spectra for the complex analytic signal representation and for the complex amplitude envelope representation are derived, respectively, and the comparison between them is made. The influences of the truncation parameter and the beam order on the spectral shifts and on the spectral switches are illustrated. It is shown that the spectrum for the complex analytic signal representation and that for the complex amplitude envelope representation are obvious different as the pulse shorter than an optical period.

1. Introduction

In 1999 Pu and his collaborators discovered the phenomenon of spectral switch, which is the spectral shift that shows a sudden change from red shift (or blue shift) to blue shift (or red shift) [1]. It has been verified experimentally by Kandpal et al. [2,3]. Foley and Wolf pointed out that it should be regarded as a manifestation of diffraction-induced spectral changes [4]. Recently, the anomalous spectral behavior of pulsed beam diffracted at an aperture has attracted considerable attentions [5–12]. Most of the previous work have been focused on the pulsed Gaussian beam. Since 2003, Cai et al. [13] introduced a convenient theoretical model named hollow Gaussian beam (HGB) to describe dark-hollow beams, the propagation of HGBs have been investigated extensively [14–18]. Xu and Lü [19] have investigated the spatiotemporal behaviour of isodiffracting hollow Gaussian pulsed beams propagating in free space. The spectral anomalies of focused spatially fully coherent polychromatic hollow Gaussian beam at the geometrical focal plane have been studied [20]. Based on the vectorial Rayleigh–Sommerfeld formulae, the propagation properties of the vector hollow Gaussian beam through a circular aperture are studied and the integral expressions of it are derived [21]. To our knowledge, the spectral characteristic of an ultrashort pulsed hollow Gaussian beam passing through a circular aperture has not been studied. Usually, both the complex amplitude envelope (CAE) representation and the complex analytic signal (CAS) representation can be used to describe the pulsed beams. However, for an ultrashort pulsed beam, the CAS representation should be adopted to describe the propagation of the beam. The spectra for the CAE and the CAS representation of ultrashort pulsed beam are different. In this paper, we will theoretically investigate this problem in details. The paper is organized as follows. In Section 2, based on vectorial Rayleigh diffraction integral, we will derive the analytical expressions of the spectral intensity for the CAE and the CAS representations. Based on analytical expressions, the numerical calculations and discussions are presented in Section 3. Finally, a conclusion is made in Section 4.

2. The analytical expression of the spectral intensity

We consider the case in which $\boldsymbol{E}(x,y,z,t)$ is known at the plane $z=0$ and propagates in the region $z>0$. For a pulsed hollow Gaussian beam having its beam waist at the input plane $z=0$, $\boldsymbol{E}_0(x_0,y_0,0,t)$ can be written as:

$$\boldsymbol{E}_0(x_0,y_0,0,t)=E_{0x}(x_0,y_0,0,t)\hat{i}+E_{0y}(x_0,y_0,0,t)\hat{j}, \tag{1}$$

with

$$E_{0x}(x_0,y_0,0,t)=\left(\frac{x_0^2+y_0^2}{w_0^2}\right)^n\exp\left(-\frac{x_0^2+x_0^2}{w_0^2}\right)A(t), \tag{2}$$

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$$E_{0y}(x_0, y_0, 0, t) = 0. \tag{3}$$

where, n is the beam order of HGB, w_0 is related to the spot size and is assumed to be frequency independent, $\hat{i}$ and $\hat{j}$ denote unit vectors in the x_0 and y_0 directions, respectively. $A(t)$ governs the pulse shape. The spectrum $\boldsymbol{U}_0(x_0, y_0, 0, \omega)$ with a frequency ω of the pulse can be obtained by Fourier transform:

$$\boldsymbol{U}_0(x_0, y_0, 0, t) = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \boldsymbol{E}_0(x_0, y_0, 0, t) \exp(-i\omega t) \mathrm{d}t \tag{4}$$

and can be expressed as:

$$\boldsymbol{U}_0(x_0, y_0, 0, \omega) = U_{0x}(x_0, y_0, 0, \omega)\hat{i} + U_{0y}(x_0, y_0, 0, \omega)\hat{j}, \tag{5}$$

where,

$$U_{0x}(x_0, y_0, 0, \omega) = \left(\frac{x_0^2 + y_0^2}{w_0^2}\right)^n \exp\left(-\frac{x_0^2 + x_0^2}{w_0^2}\right) f(\omega), \tag{6}$$

$$U_{0y}(x_0, y_0, 0, \omega) = 0, \tag{7}$$

here, $f(\omega)$ is the Fourier transform of the $A(t)$.

A circular aperture with the radius a is assumed to be located at the incident plane $z = 0$. According to the vectorial Rayleigh diffraction integral, the field at the z plane behind the circular aperture can be obtained:

$$U_x(\boldsymbol{r}, \omega) = -\frac{1}{2\pi} \int \int_{-\infty}^{\infty} T(x_0, y_0) U_{0x}(x_0, y_0, 0, \omega) \frac{\partial G(\boldsymbol{r}, \boldsymbol{r}_0)}{\partial z} \mathrm{d}x_0 \mathrm{d}y_0, \tag{8}$$

$$U_y(\boldsymbol{r}, \omega) = -\frac{1}{2\pi} \int \int_{-\infty}^{\infty} T(x_0, y_0) U_{0y}(x_0, y_0, 0, \omega) \frac{\partial G(\boldsymbol{r}, \boldsymbol{r}_0)}{\partial z} \mathrm{d}x_0 \mathrm{d}y_0, \tag{9}$$

$$\begin{aligned} U_z(\boldsymbol{r}, \omega) = \frac{1}{2\pi} \int \int_{-\infty}^{\infty} T(x_0, y_0) \Bigg[& U_{0x}(x_0, y_0, 0, \omega) \frac{\partial G(\boldsymbol{r}, \boldsymbol{r}_0)}{\partial z} \\ & + U_{0y}(x_0, y_0, 0, \omega) \frac{\partial G(\boldsymbol{r}, \boldsymbol{r}_0)}{\partial z} \Bigg] \mathrm{d}x_0 \mathrm{d}y_0, \end{aligned} \tag{10}$$

where, $\boldsymbol{r}_0 = x_0\hat{i} + y_0\hat{j}$, $\boldsymbol{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $\hat{k}$ is the unit vector in the z direction,

$$T(x_0, y_0) = \begin{cases} 1, & x_0^2 + y_0^2 \leq a^2, \\ 0, & x_0^2 + y_0^2 > a^2, \end{cases} \tag{11}$$

denotes the window function of a circular aperture located at plane $z = 0$, and

$$G(\boldsymbol{r}, \boldsymbol{r}_0) = \frac{\exp(ik\,|\boldsymbol{r} - \boldsymbol{r}_0|)}{|\boldsymbol{r} - \boldsymbol{r}_0|}, \tag{12}$$

is the Green function. For following calculation, the Green function can be approximated as follows:

$$G(\boldsymbol{r}, \boldsymbol{r}_0) \approx \frac{1}{r} \exp\left[ik\left(r + \frac{x_0^2 + y_0^2 - 2xx_0 - 2yy_0}{2r}\right)\right], \tag{13}$$

where, $k = \omega/c$ is the wave number, and c is the velocity of light in vacuum, $r = (x^2 + y^2 + z^2)^{1/2}$. By using Eqs. (11) and (13) in Eqs. (8)–(10) and performing the integration, we obtain the result:

$$\begin{aligned} U_x(x, y, z, \omega) = & -i(-1)^n \frac{\omega z \exp(ikr)}{cr^2 w_0^{2n}} f(\omega) \sum_{s=0}^{\infty} \frac{1}{2^{2s+1}(s!)^2} \left(\frac{\omega}{c}\right)^{2s} \left(\frac{\rho}{r}\right)^{2s} \frac{1}{\beta^{s+n+1}} \\ & \times [\Gamma(1 + s + n, -a^2\beta) - (s + n)!], \end{aligned} \tag{14}$$

$$U_y(x, y, z, \omega) = 0, \tag{15}$$

$$\begin{aligned} U_z(x, y, z, \omega) = & \; i(-1)^n \frac{\omega x \exp(ikr)}{cr^2 w_0^{2n}} f(\omega) \sum_{s=0}^{\infty} \frac{1}{2^{2s+1}(s!)^2} \left(\frac{\omega}{c}\right)^{2s} \left(\frac{\rho}{r}\right)^{2s} \frac{1}{\beta^{s+n+1}} \\ & \times [\Gamma(1 + s + n, -a^2\beta) - (s + n)!], \\ & + (-1)^n \frac{\omega x \exp(ikr)}{cr^3 w_0^{2n}} f(\omega) \sum_{s=0}^{\infty} \frac{1}{2^{2s+1} s!(s + 1)!} \left(\frac{\omega}{c}\right)^{2s+1} \left(\frac{\rho}{r}\right)^{2s+1} \\ & \times \frac{1}{\beta^{s+n+2}} [\Gamma(2 + s + n, -a^2\beta) - (s + n + 1)!], \end{aligned} \tag{16}$$

where, $\Gamma(\cdot)$ is the incomplete gamma function, $\rho = (x^2 + y^2)^{1/2}$ and

$$\beta = i\frac{\omega}{2cr} - \frac{1}{w_0^2}. \tag{17}$$

If adapted the CAS representation and the CAE representation, respectively, $f(\omega)$ take different form for the same pulse. In order to know the differences between the spectrum for the CAS representation and that for the CAE representation, we take Gaussian pulse as an example. For the CAE representation, the $A(t)$ takes the following form:

$$A(t) = \exp\left(-\frac{a_g^2 t^2}{T^2}\right) \exp(i\omega_c t), \tag{18}$$

where, $a_g = 2(\ln 2)^{1/2}$, ω_c is carry frequency, T is the pulse duration. $m = T/T_c$ is the number of optical cycle for the pulse, where T_c is the time period corresponding to the carry frequency. Taking the Fourier transform of $A(t)$, we obtained:

$$f(\omega) = \frac{T}{\sqrt{2}a_g} \exp\left[-\frac{T^2(\omega - \omega_c)^2}{4a_g^2}\right]. \tag{19}$$

For the CAS representation, the form of $f(\omega)$ can be expressed as follows:

$$f(\omega) = \frac{T}{2\sqrt{2}a_g} \left\{ \exp\left[-\frac{T^2(\omega - \omega_c)^2}{4a_g^2}\right] + \exp\left[-\frac{T^2(\omega + \omega_c)^2}{4a_g^2}\right] \right\}. \tag{20}$$

The spectral intensity of the diffraction field can be obtained from Eqs. (14)–(16):

$$I(x, y, z, \omega) = \left|U_x(x, y, z, \omega)\right|^2 + \left|U_y(x, y, z, \omega)\right|^2 + \left|U_z(x, y, z, \omega)\right|^2. \tag{21}$$

Eq. (21) is main result of this work, which provides a general approximate expression for the spectral intensity of a nonparaxial vectorial hollow Gaussian pulsed beam diffracted at a circular aperture. The effects of vectorial and nonparaxial nature on spectral anomalies of pulsed Gaussian beam passing through a hard-edged aperture have been analyzed in details [11]. We will not discuss the effects of vectorial and nonparaxial nature in this paper any more.