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Synchronization of time varying delayed complex networks via impulsive control

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ABSTRACT

In this paper, we investigate the problem of synchronization for the time varying delayed complex dynamical networks via impulsive control method, several sufficient synchronization conditions are given, and we consider the impulsive control matrices are time varying delayed matrices. Furthermore, we found impulsive control does not always play an active role in synchronization although impulsive control strategy is cheaper and simpler than other control strategy. Finally, numerical simulations are also given to demonstrate the effectiveness of the proposed schemes.

1. Introduction

As is known to all, complex dynamical networks widely exist in the real world, including food webs, communication networks, social networks, power grids, cellular networks, world wide web, metabolic systems, disease transmission networks, and so on [1–5]. As obstructions to the transmission of signals are inevitable in a biological neural network, in an epidemiological model, in a communications network, or in an electrical power grid, one important consideration in practical networks is the existence of time delays. Apart from complex systems coupling delays [6–14], complex systems are susceptible to sudden surges in their flows called impulses. Due to its many potential practical applications, the problem of impulsive control and synchronization of complex dynamical networks has been extensively investigated in various fields of science and engineering in recent years [15–25]. In practice, the impulsive matrices are usually chosen to be constant matrices. The question naturally arises as to whether it is possible to achieve synchronization for the time varying delayed impulsive matrices. Moreover, the existing synchronization of time varying delay dynamical networks, most offered LMI synchronization conditions, which is inconvenient to calculate impulsive interval $t_{k+1} - t_k$. In this paper, we consider the problem of synchronization for the time varying delayed complex dynamical networks via the impulsive control time varying delayed matrices. Although impulsive control is an artificial control strategy which is cheaper and simpler to operate compared with other control strategy, impulsive control does not always play an active role in synchronization.

The rest of this work is organized as follows: Section 2 gives the problem formulation. Section 3 gives impulsive synchronization criterion. Section 4 gives illustrative example. Section 5 gives the conclusion of this paper.

2. Problem formulation

In this paper, we consider the following time varying delayed complex networks with perturb function

$$dx^i(t) = \left[f(x^i(t)) + \sum_{j=1}^{N} a_{ij} x^j(t) + \sum_{j=1}^{N} b_{ij}(x^j(t - \tau(t)) \right] dt + \sigma(t, x^i(t)), \tag{1}$$

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where $x^i = (x_1, x_2, \ldots, x_n)^T \in R^n$, $f: R^n \to R^n$ standing for the activity of an individual subsystem is a vector value function. $A = (a_{ij})_{N\times N} \in R^{N\times N}$ and $B = (b_{ij})_{N\times N} \in R^{N\times N}$ are the coupling matrices, and a_{ij} and b_{ij} are the weight or coupling strength. If there exists a link from node i to $j(i \neq j)$, then $a_{ij} \neq 0$ and $b_{ij} \neq 0$. Otherwise, $a_{ij} = 0$ and $b_{ij} = 0$. The time-varying delayed $\tau(t)$ satisfying that $\dot{\tau}(t) \leq \tau < 1$, where τ is constant. $\sigma : R^n \times R^+ \to R^{n\times m}$ is the perturb function.

In the paper, we have the following mathematical preliminaries:

Assumption 1. We also assume that f is Lipschitz with respect to its argument i.e.

$$\left|f(y^i(t)) - f(x^i(t))\right| \leq \eta e^i(t),\ \eta \in R.$$

Assumption 2. There exists nonnegative constant h^i, such that

$$\sigma(y^i(t), t) - \sigma(x^i(t), t) = h^i e^i(t) \leq He^i(t),$$

where $H = \max(h^i)$.

Lemma 1. [26] for any vectors $x, y \in R^m$ and positive definite matrix $Q \in R^{m\times m}$, the following matrix inequality holds: $2x^T y \leq x^T Qx + y^T Q^{-1} y$. If not specified otherwise, inequality $Q > 0$ ($Q < 0$, $Q \geq 0$, $Q \leq 0$) means Q is a positive (or negative, or semi-positive, or semi-negative) definite matrix.

3. Impulsive synchronization criterion

We will investigate impulsive synchronization of the complex network with perturb function in this section. Under impulsive effects, two delayed coupled complex networks can be described by

$$dx^i(t) = \left[f(x^i(t)) + \sum_{j=1}^{N} a_{ij} x^j(t) + \sum_{j=1}^{N} b_{ij}(x^j(t-\tau(t))\right] dt + \sigma(t, x^i(t)),\ t \neq t_k, \tag{2}$$

$$\Delta x^i(t_k) = x^i(t_k^+) - x^i(t_k^-) = -(B^{ik} + C^{ik}(t-\tau(t)))e^i(t_k^-),\ t = t_k, \tag{3}$$

$$dy^i(t) = \left[f(y^i(t)) + \sum_{j=1}^{N} a_{ij} y^j(t) + \sum_{j=1}^{N} b_{ij}(y^j(t-\tau(t))\right] dt + \sigma(t, y^i(t)),\ t \neq t_k, \tag{4}$$

$$\Delta y^i(t) = y^i(t_k^+) - y^i(t_k^-) = (B^{ik} + C^{ik}(t-\tau(t)))e^i(t_k^-),\ t = t_k, \tag{5}$$

where $y = (y_1, y_2, \ldots, y_n)^T \in R^n$. $C^{ik}(t-\tau(t))$ are time varying delayed matrices, $\left\|C^{ik}(t-\tau(t))\right\| \leq 1$. $\sigma : R^n \times R^+ \to R^{n\times m}$ is the perturb function.

Suppose the synchronization errors $e^i(t) = y^i(t) - x^i(t)$, then we have the following error dynamical equations:

$$de^i(t) = \left[f(y^i(t)) - f(x^i(t)) + \sum_{j=1}^{N} a_{ij} e^j(t) + \sum_{j=1}^{N} b_{ij} e^j(t-\tau(t))\right] dt + h^i e^i,\ t \neq t_k, \tag{6}$$

$$\Delta e^i(t_k) = e^i(t_k^+) - e^i(t_k^-) = 2(B^{ik} + C^{ik}(t-\tau(t)))e^i(t_k^-),\ t = t_k, \tag{7}$$

where $e(t_k^+) = \lim_{t\to t_k^+} e(t)$, $e(t_k) = \lim_{t\to t_k^-} e(t) = e(t_k^-)$.

Theorem 1. Under Assumptions 1 and 2, if there exists a constant $\theta \geq 1$, then the complex dynamical networks (2) and (4) can realize impulsive synchronization using the following form

$$\ln \theta\rho_k + 2\left(\eta + N\mu_1 + \frac{1}{2}\frac{N\mu_2^2}{1-\tau} + \frac{1}{2} + H\right)(t_{k+1} - t_k) \leq 0,$$

where $\left|a_{ij}\right| \leq \mu_1 \in R$, $\left|b_{ij}\right| \leq \mu_2 \in R$, $\rho_k = \lambda_{\max}[(3I + 2B^{ik})^T (3I + 2B^{ik})]$.

Proof. We choose a non-negative function as

$$V(t) = \frac{1}{2}\sum_{i=1}^{N}(e^i(t))^T e^i(t) + \frac{1}{2}\sum_{i=1}^{N}\int_{t-\tau(t)}^{t}(e_i(s))^T e_i(s)ds \tag{8}$$

Then the differentiation of V along the trajectories of (6) is

$$\dot{V}(t) = \sum_{i=1}^{N}(e^i(t))^T \left[f(y^i(t)) - f(x^i(t)) + \sum_{j=1}^{N} a_{ij} e^j(t) + \sum_{j=1}^{N} b_{ij} e^j(t-\tau(t))\right]$$

$$+ \sum_{i=1}^{N} h^i (e^i(t))^T e^i(t) + \frac{1}{2}\sum_{i=1}^{N}(e^i(t))^T e^i(t) - \frac{1}{2}(1-\dot{\tau}(t))\sum_{i=1}^{N}(e^i(t-\tau(t)))^T e^i(t-\tau(t))$$