Optik 125 (2014) 3825–3828

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

The visibility analysis of correlated imaging based on the coherent mode representation

Yanfeng Bai [a,*], Wenxing Yang [a], Dingan Han [b], Xiaoqiang Yu [a]

[a] Department of Physics, Southeast University, Nanjing 210096, China
[b] Department of Physics, Foshan University, Foshan 528000, China

ARTICLE INFO

Article history:
Received 17 July 2013
Accepted 9 January 2014

Keywords:
Correlated imaging
The coherent mode representation
Intensity fluctuation correlation

PACS:
42.50.Dv
42.50.Ar
42.30.Va

ABSTRACT

The visibility of ghost-interference patterns is investigated firstly based on the coherent-mode representation theory. We find that the intensity correlation function can be changed from the usually two-dimensional integral representation to a new one-dimensional summation representation. During the process of analyzing the effects from the light source's properties and the transmission area of the object imaged on the imaging visibility, it is shown that by compared with the results from the integral representation, the coherent mode representation is quite useful to understand more clearly the whole process of correlated imaging.

1. Introduction

Correlated imaging has attracted much attention in recent years. To retrieve the information about an unknown object, the technique takes advantage of the quantum correlation between a pair of photons generated by parametric down conversion (PDC) [1–6]. In a correlated imaging system, the photons of a pair are spatially separated and travel through two different imaging systems, the spatial information of the object inserted into one of the imaging systems can be non-locally recovered by analyzing the conditional detection probability of a photodetector placed in the other imaging system. Notice that the first two-photon imaging experiment was performed with entangled photons [2]. While a great deal of attention has been put on the possibility of performing correlated imaging with classically thermal source now [7–16]. However, the visibility of classical correlated imaging is limited owing to the presence of an intrinsic background term in the second-order correlation function. To overcome the obstacles, some effects turn to correlated imaging with high-order intensity correlation [17–22]. In addition, a computational ghost-imaging [23] which can be used to optical encryption [24] was reported.

Very recently, the coherent-mode representation of partially coherent field which is quite different from the usually two-dimensional integral representation, has been proposed to analyze correlated imaging, the results showed that this method is particularly suitable for evaluating the imaging quality [25]. In this paper, motivated by this work, we firstly use the coherent-mode representation theory to investigate the imaging visibility in correlated imaging with partially coherent field. It is shown that the visibility of ghost-interference patterns can be analyzed by the distribution of the eigenvalue of the coherent-mode representation of the source and the distribution of the decomposition coefficient of the object imaged. Based on the results, we can understand more clearly the effects from light's properties and the object transmissive area.

2. Model and equations

A typical system for correlated imaging is shown in Fig. 1. The source is divided into two beams by a beam splitter, they travel on their respective paths to be detected at spatially separated detection systems. In reference arm there is not usually any object, the beam simply propagates to the detector D_2, the reference detector spatially resolves the light fluctuations, as for example an array of pixel detectors. In test arm, the beam usually propagates to the object imaged, and then, after propagation, it travels to the detector D_1 which is a pointlike detector, in any case D_1 gives no

* Corresponding author. Tel.: +86 25 52090605-8204; fax: +86 25 52090605-8204.
E-mail address: yanfengbai@gmail.com (Y. Bai).

http://dx.doi.org/10.1016/j.ijleo.2014.01.178
0030-4026/

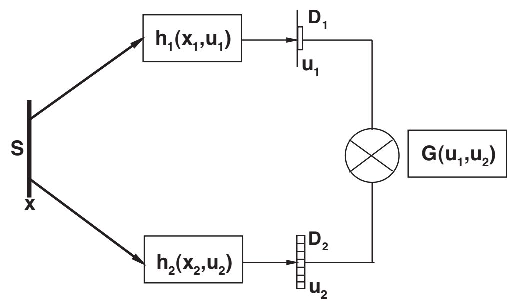

Fig. 1. A simplified scheme for correlated imaging.

information on the object spatial distribution. The two arm systems are characterized by their response functions $h_2(x_2, u_2)$ and $h_1(x_1, u_1)$, respectively.

In the thermal case, we can retrieve the information of the object imaged by measuring the spatial correlation function of the intensities detected by D_1 and D_2, which may be recorded with the coincidence rate [26–28].

$$G^{(2)}(u_1, u_2) = \langle E(u_1)E(u_2)E^*(u_2)E^*(u_1)\rangle$$
$$= \langle I_1(u_1)\rangle\langle I_2(u_2)\rangle + G(u_1, u_2), \quad (1)$$

where $\langle I_i(u_i)\rangle$ $(i = 1, 2)$ is the intensity distribution at the ith detector, $G(u_1, u_2)$ is the correlation function of intensity fluctuations depending on both paths, and the first term $\langle I_1\rangle\langle I_2\rangle$ which only contributes a background cannot be used to implement correlated imaging. So all the object information is contained in the second term, i.e., the intensity fluctuation correlation. Here we have [9]

$$\begin{cases} G(u_1, u_2) & = \langle \Delta I_1(u_1)\Delta I_2(u_2)\rangle \\ & = \left| \int \Gamma(x_1, x_2)h_1(x_1, u_1)h_2^*(x_2, u_2)dx_1dx_2 \right|^2, \\ \langle I_i(u_i)\rangle & = \int \Gamma(x_1, x_2)h_i(x_1, u_i)h_i^*(x_2, u_i)dx_1dx_2, \end{cases} \quad (2)$$

where $\Gamma(x_1, x_2)$ is the second-order correlation function of the source, and $\Delta I_i(u_i) = I_i(u_i) - \langle I_i(u_i)\rangle$.

Following along the lines of Ref. [25], let us consider the second-order coherence theory of optical fields [29], and substitute the second-order correlation function of the source which is expressed in the coherent-mode representation into Eq. (1). The intensity correlation function given in Eq. (1) can be rewritten as

$$G^{(2)}(u_1, u_2) = \sum_n \beta_n \left|f_n(u_1)\right|^2 \sum_n \beta_n \left|g_n(u_2)\right|^2$$
$$+ \left| \sum_n \beta_n f_n^*(u_1) g_n(u_2) \right|^2. \quad (3)$$

The significant point about Eq. (3) is that both the background term and the intensity fluctuation correlation term are changed from the usually two-dimensional integral representation to a new one-dimensional summation representation. Note that the first term of Eq. (1) is ignored in Ref. [25], so only imaging quality was discussed. Here we consider this term, and change it into the one-dimensional summation representation, so we can investigate the imaging visibility based on Eq. (3).

Here we consider a partially coherent Gaussian-Schell model (GSM) source, the second-order correlation function in the GSM source plane has [29]

$$\Gamma(x_1, x_2) = G_0 \exp\left[-\frac{x_1^2 + x_2^2}{4\sigma_l^2} - \frac{(x_1 - x_2)^2}{2\sigma_g^2} \right],$$

where G_0 is a normalized constant, σ_l is the source's transverse size, and σ_g is the transverse coherence width of the source. To apply the coherent-mode theory to correlated imaging, we should firstly give the coherent-mode representation of the GSM source, which has been shown in Ref. [29]. The corresponding eigenvalue and eigenfunction have

$$\beta_n = \left(\frac{\pi}{a+b+c} \right)^{1/2} \left(\frac{b}{a+b+c} \right)^n,$$

$$\phi_n(x) = \left(\frac{2c}{\pi} \right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n(x\sqrt{2c})e^{-cx^2}, \quad (4)$$

where $a = 1/4\sigma_l^2$, $b = 1/2\sigma_g^2$, and $c = \sqrt{a^2 + 2ab}$. $H_n(x)$ are the Hermite polynomials. Here we choose a simple 2-f imaging system [8] to discuss our results. By substituting the corresponding impulse response functions into the coherent mode representation [25], the conditional intensity correlation can be expressed as

$$G^{(2)}(u_1 = 0, u_2) = \sum_n \beta_n \left|f_n(0)\right|^2 \sum_n \beta_n \left| \Phi_n\left(\frac{2\pi u_2}{\lambda f} \right) \right|^2$$
$$+ \left| \sum_n \beta_n f_n^*(0) \Phi_n\left(\frac{2\pi u_2}{\lambda f} \right) \right|^2. \quad (5)$$

where $\Phi_n(q)$ are the Fourier transform of $\phi_n(x)$, and $f_n(0)$ can be looked upon as the decomposition coefficients of the object [25]. It should be noted that the first term of Eq. (5) does not exist in the results given in Ref. [25].

3. The numerical results

From the conclusions in Ref. [25], the imaging quality crucially depends on the distribution of the eigenvalues β_n of the coherent-mode presentation of the source and the decomposition coefficients $f_n(0)$ of the object imaged. In the following, we will attempt to apply this method to analyzing the changes of the imaging visibility. Firstly let us investigate the effects of the light source's transverse size and coherence width, which were theoretically discussed under the usual integral representation [26] and then experimentally implemented [30]. During the process, a double-slit with the slit width 0.07 mm and the distance between two slits 0.16 mm is chosen as the object imaged. From Eq. (5), we can easily obtain the normalized intensity correlation function $G^{(2)}(u_1 = 0, u_2)$. Fig. 2 shows the distribution of the eigenvalues β_n of the coherent-mode representation of the source and the decomposition coefficients $f_n(0)$ of the object, the corresponding intensity correlation function which is normalized by its maximum value is also presented. Here we choose the parameters as $\sigma_l = \sqrt{1/4a} = 3.5$ mm, $\sigma_g = \sqrt{1/2b} = 88.4$ µm, and $n = 50$. It is interesting to note that $f_n(0)$ has a relative narrow distribution when comparing with the distribution of β_n, and the visibility of the ghost-interference pattern is quite small.

Now, the most obvious question is whether the changes of the visibility can be understood by the varieties of the distribution of β_n and $f_n(0)$. To this end, we set $\sigma_l = 1$ mm, but keep other parameters unchanged in Fig. 3. It is shown that by comparing Fig. 3(c) with Fig. 2(c), a decrease of the source's transverse size will lead to an