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Quantum polarization fluctuation of a multi-Gaussian Shell-model beam in a slant turbulent channel

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ABSTRACT

The influence of atmospheric turbulence on the quantum polarization fluctuations of multi-Gaussian Schell-mode (MGSM) beams is studied in detail. An analytical formula for the quantum degree of polarization of a MGSM beam propagating in a slant turbulent channel is derived. Our results show that the degree of polarization of a MGSM beam is affected more by the atmospheric turbulence than that of a GSM beam. The numerical simulations also show that a MGSM beam with higher photon-number level, shorter wavelength, bigger transverse beam width is less affected by the turbulence.

1. Introduction

Polarization is one of the most important properties of light with a large number of applications, both in the quantum and classical domains. It is often chosen as the variable for encoding quantum information in recent experiments, which have demonstrated the feasibility of quantum key distribution in free-space channels [1–6]. Quantum light propagation in the channels between earth-based stations and between satellites and earth-based stations will be affected by turbulent atmosphere. In recent years, many studies related to transfer quantum polarization information through the turbulent atmosphere have been published [7–10]. Nonclassical polarization properties of quantum light are thus required to study in the context of quantum theory, and they can be described by the quantum Stokes operators completely [11–13].

Just recently, we develop a theoretical model for the quantum polarization fluctuations based on the quantum Stokes operators and the extended Huygens–Fresnel principle and study the quantum polarization fluctuations of Gaussian beams in turbulent atmosphere [14]. More recently, a novel beam named multi-Gaussian Schell-mode (MGSM) beam whose far field intensity distribution has a flat-topped beam profile was introduced [15], and the evolution properties of the intensity distribution, the scintillation index, and the second-order moments of a MGSM beam in turbulent atmosphere were reported [16–18]. In this paper, our aim is to derive the analytical formula for the quantum degree of polarization of a MGSM beam propagating in a slant turbulent atmosphere channel, and explore the influence of atmospheric turbulence on the quantum polarization fluctuations of MGSM beams.

2. Theoretical model

The quantum polarization state of a light field can be succinctly described by the quantum Stokes operators [11–13]. The polarization changes of linear polarized light in a turbulent atmosphere can be easily accounted by the quantum degree of polarization [14]

$$P = \sqrt{\frac{\langle s_0\rangle_{at}}{\langle s_0\rangle_{at}+2}} \tag{1}$$

where $s_0 = \langle \xi \left| \hat{a}_x^{\dagger}(\boldsymbol{\rho},\mathbf{q})\hat{a}_x(\boldsymbol{\rho},\mathbf{q}) \right| \xi\rangle$ is the Stokes parameter, $\langle\cdots\rangle_{at}$ denote average over the ensemble of the turbulent atmosphere, here $\hat{a}_x^{\dagger}(\boldsymbol{\rho},\mathbf{q})$ and $\hat{a}_x(\boldsymbol{\rho},\mathbf{q})$ denote the photon creation and annihilation operators of polarization modes x in receiving plane, and $\boldsymbol{\rho}$ is the transverse position vector and $\mathbf{q}$ ($|\mathbf{q}| = k = 2\pi/\lambda$) is the transverse wave-vector of a photon.

In the case of paraxial propagation, the Stokes parameters of a beam propagation through atmospheric turbulence in the z plane can be expressed as [14]

$$\langle s_0\rangle_{at} = n_\tau \left(\frac{k}{2\pi z}\right)^2 \int\int \mathrm{d}^2\boldsymbol{\rho}'\mathrm{d}^2\boldsymbol{\rho}'' W(\boldsymbol{\rho}',\boldsymbol{\rho}'',0)$$

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$$\times \exp\left\{-\frac{ik}{2z}\left[\left(\boldsymbol{\rho}-\boldsymbol{\rho}'\right)^2-\left(\boldsymbol{\rho}-\boldsymbol{\rho}''\right)^2\right]\right\}$$
$$\times \langle \exp\left[\psi^*\left(\boldsymbol{\rho},\boldsymbol{\rho}',z\right)+\psi\left(\boldsymbol{\rho},\boldsymbol{\rho}'',z\right)\right]\rangle_{at}, \quad (2)$$

where $n_\tau = \tau n_0$ is the number of transmissive receiving photon, τ is transmittance of channel, $\hat{n}_0(\mathbf{q}) = \hat{a}^\dagger_{0x}(\mathbf{q})\hat{a}_{0x}(\mathbf{q})$ is the initial number operator in source plane ($z=0$), $\hat{a}_{0x}(\mathbf{q})$ is the photon annihilation operator. A photon eignstate is a pure state which is described by wave function $|\xi(\boldsymbol{\rho})\rangle$ and $\hat{n}_0(\mathbf{q})|\xi(\boldsymbol{\rho},\mathbf{q})\rangle = n_0(\mathbf{q})$. $\psi(\boldsymbol{\rho},\boldsymbol{\rho}',z) = \chi(\boldsymbol{\rho},\boldsymbol{\rho}',z) + is(\boldsymbol{\rho},\boldsymbol{\rho}',z)$ describes the effects of the atmospheric turbulence on the propagation of a spherical wave, here terms $\chi(\boldsymbol{\rho},\boldsymbol{\rho}',z)$ and $s(\boldsymbol{\rho},\boldsymbol{\rho}',z)$ account for the stochastic log-amplitude and phase fluctuations, respectively, imposed by atmospheric turbulence. $W(\boldsymbol{\rho}',\boldsymbol{\rho}'',0)$ is the cross-spectral density function of a MGSM beam in source plane and can be expressed as [16]

$$W(\boldsymbol{\rho}',\boldsymbol{\rho}'',0) = \frac{1}{C_0}\exp\left(-\frac{\boldsymbol{\rho}'^2+\boldsymbol{\rho}''^2}{4\sigma^2}\right)\sum_{m=1}^{M}\binom{M}{m}\frac{(-1)^{m-1}}{m}$$
$$\times \exp\left[-\frac{(\boldsymbol{\rho}'-\boldsymbol{\rho}'')^2}{2m\delta^2}\right], \quad (3)$$

where $C_0 = \sum_{m=1}^{M}\frac{(-1)^{m-1}}{m}\binom{M}{m}$ is the normalization factor, $\binom{M}{m}$ denotes a binomial coefficient, σ is the transverse beam width of the source, and δ is the transverse correlation width. When $M=1$, the cross-spectral density function of a MGSM beam reduces to that of a GSM beam with δ being the transverse correlation width.

Taking into account the Kolmogorov spectrum and a quadratic approximation of the 5/3 power law for Rytovs phase structure function, the ensemble average term in Eq. (2) can be expressed as [19]

$$\langle \exp\left[\psi^*\left(\boldsymbol{\rho},\boldsymbol{\rho}',z\right)+\psi\left(\boldsymbol{\rho},\boldsymbol{\rho}'',z\right)\right]\rangle_{at}$$
$$= \exp\left\{-\left[\left(\boldsymbol{\rho}'-\boldsymbol{\rho}''\right)^2\right]^{5/6}/\rho_0^{5/3}\right\}, \quad (4)$$

where ρ_0 is the spatial coherence radius of a spherical wave propagation in turbulent atmosphere and can be expressed as [20]

$$\rho_0 = \left(0.545k^2 z\int_0^1 C_n^2(\xi z\cos\theta)d\xi\right)^{-3/5}, \quad (5)$$

where $C_n^2(z\cos\theta)$ is a generalized refractive-index structure parameter, which is altitude dependent and is given by [21]

$$C_n^2(z\cos\theta) = 8.148\times10^{-56}v^2(z\cos\theta)^{10}\exp(-z\cos\theta/1000)$$
$$+2.7\times10^{-16}\exp(-z\cos\theta/1500) + C_n^2(0)\exp(-z\cos\theta/100), \quad (6)$$

where $z\cos\theta = h$ is altitude, v is the rms wind speed (a typical value is 21 m/s), $C_n^2(0)$ is the refractive index structure parameter at the ground, and θ is the zenith angle of communication channel.

Substituting Eqs. (3) and (4) into Eq. (2) and after tedious integral calculations, we obtain the average Stokes parameter s_0 of turbulent ensemble

$$\langle s_0\rangle_{at} = \frac{n_\tau k^2}{4z^2 C_0}\sum_{m=1}^{M}\binom{M}{m}\frac{(-1)^{m-1}}{mT_1}\exp\left(-\frac{T_2}{T_1}\rho^2\right), \quad (7)$$

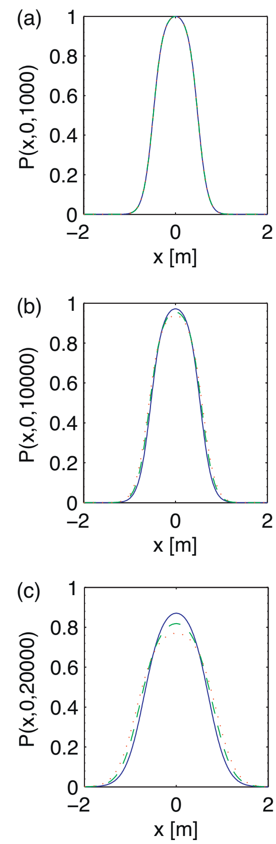

Fig. 1. Transverse cross section of the degree of polarization $P(x, 0, z)$ for different values of propagation distance z: (a) 1 km, (b) 10 km, and (c) 20 km. Solid, dashed and dotted curves denote propagation beam with different values of $M = 1$, 10, and 50, respectively. Other calculation parameters are: $C_n^2(0) = 1.7\times10^{-14}\,\mathrm{m}^{-2/3}$, $\sigma = 0.2$ m, $\delta = 0.001$ m, $\theta = \pi/6$, $\lambda = 632$ nm, $n_\tau = 10$.

where

$$T_1 = \frac{1}{16\sigma^4} + \frac{1}{2\sigma^2}\left(\frac{1}{2m\delta^2}+\frac{1}{\rho_0^2}\right) + \frac{k^2}{4z^2}, \quad T_2 = \frac{k^2}{8\sigma^2 z^2}.$$

Substituting Eq. (7) into Eq. (1), we can obtain the formula for the degree of polarization $P(\boldsymbol{\rho}, z)$ of a MGSM beam propagating in a slant turbulent atmosphere channel.

3. Numerical results and analysis

In this section, we study the numerical results of the degree of polarization of MGSM beams in a slant turbulent channel by using the formula derived in the above section.

In Fig. 1 we show the transverse cross sections of the degree of polarization of a MGSM beam at several selected distances and for several values of index M. All curves preserve Gaussian shape, and as the propagation distance grows, the transverse profiles of the degree of polarization depend on the index M gradually: the larger the value of M is, the smaller its maximum height and the flatter its profile becomes. Moreover, we can see from Fig. 1 that the degree of polarization decreases gradually with increase in the value of propagation distance z.

Fig. 2 gives the degree of polarization at the beam axis for different values of index M and for different values of transverse correlation width δ. It can be seen from Fig. 2 that the degree of polarization decreases monotonically with the increasing of the propagation distance, and for a fixed z, it depends on the index M and the coherence length δ: the larger the value of M or the smaller the value of δ is, the smaller its value at the beam axis becomes. We