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Modulation instability by intense laser beam in magnetized plasma

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ABSTRACT

Modulation instability of an intense right-hand elliptically polarized laser beam propagating through magnetized plasma is investigated by a new method. The nonlinear dispersion relation, in which the relativistic and ponderomotive nonlinearities are taken into account, is obtained for the laser radiation in magnetized plasma by the Lorentz transformation. The Karpman equation is firstly generalized to the case of three dimensions with three field components. When the nonlinear frequency shift of the electromagnetic field in plasma is involved, the nonlinear evolution equation for the slowly varying envelope of the laser field is obtained. Thus, modulation instability of the intense laser beam in magnetized plasma is studied and the temporal growth rate of the instability is derived. The analysis shows that the peak growth rate of self-modulation instability is increased due to the axial magnetization of plasma. It is also shown that the growth rate of modulation instability is increased significantly near the critical surface in a laser–plasma.

1. Introduction

Laser–plasma interactions can give rise to a variety of relativistic and nonlinear effects, including self-modulation, self-focusing, parametric instabilities, stimulated Brillouin and Raman scattering [1–4]. These physical phenomena occur in many events such as inertial confinement fusion (ICF), X-ray generation, laser-driven acceleration and optical harmonic generation [5–7]. When an incident laser beam propagates through plasma, there is modulational interaction between transverse plasmons and driven ion-acoustic wave. It leads to a nonlinear shift in the plasma frequency in such a way that the amplitude of the transverse pump wave becomes modulated, and the modulation instability occurs. The modulation instability is significant because the growth rate of the instability is directly proportional to the finite amplitude of the pump wave.

The interaction between intense laser pulse and magnetized plasma is an important area of study. When a beam interacts with plasma, intense transverse as well as axial magnetic fields are generated [8–10]. An important application where either self-generated or external magnetic fields in plasma can play a significant role is the fast ignitor scheme in ICF [11]. Spontaneously generated or applied magnetic fields significantly affect various laser plasma instabilities. When a laser beam propagates through plasma embedded in a uniform magnetic field, the plasma electron motion will be modified due to the magnetic field and will give rise to changes in the dispersion of the laser beam and nonlinear current density. The intensities of ultraintense laser pulses, in excess of 10^{18} W/cm^2, lead to relativistic electron motion in the laser field [12]. The consequent electron-mass variation can lead to modulation instability of laser pulses propagating through the plasma. Since the relativistic modulation instability arising with high laser intensities can grow so quickly in time that ion inertia prevents the ions from following along, only electron motions are involved. Modulational instability is the consequence of the interplay between nonlinearity and dispersive effects. The perturbation to the laser pulse experiences exponential growth, and this leads to beam breakup in either space or time. In nonlinear optics, modulational instability has been experimentally demonstrated in both the temporal and spatial domain [13]. The modulational instability of laser beam in plasma has been discussed in several publications [2,14–16]. The early work mostly considered one-dimensional nonlinear wave equation in which transverse variations of the laser field amplitude are neglected [11,15,17,18]. The Karpman equation for the envelope of the electric field with one polarization was

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derived from the nonlinear dispersion equation [19]. Shukla and Stenflo used the Karpman equation to investigate the nonlinear propagation of electromagnetic waves in magnetized plasmas [20]. In this paper, we generalize the Karpman equation to the case of three dimensions with three field components and derive the nonlinear equation describing the evolution of the envelope of the laser electric field. On the other hand, previous studies focused mainly on the spatial distribution of modulation instability of laser pulse in plasma [11,14]. Generally the temporal distribution of modulation instability is more important and it is intimately connected with the turbulence [21]. In this paper, we consider the arbitrary direction of low-frequency perturbations and present a unified formalism for modulation instability.

The present paper is devoted to the study of modulation instability for an intense right-hand elliptically polarized laser beam in magnetized plasma. The structure of the paper is as follows: In Section 2, the nonlinear dispersion relation for an intense right-hand elliptically polarized laser pulse in magnetized plasma is derived by means of the Lorentz transformation. In Section 3, the Karpman equation is generalized to the case of transverse field and the nonlinear evolution equation for the slowly varying envelope of the laser field is obtained theoretically. In Section 4, the modulation instability of a finite-amplitude monochromatic pump wave in magnetized plasma is analyzed and the temporal growth rate is given. In Section 5, the discussion and conclusions are presented.

2. Nonlinear dispersion relation

The analysis in this section is on the basis of the Lorentz transformation adopted in Ref. [22]. Consider an intense right-hand elliptically polarized laser beam propagating along the z direction through magnetized plasma. The electric vector of the radiation field is represented by

$$\mathbf{E} = \mathbf{E}_0(\mathbf{r}, t)\exp[i(k_0 z - \omega_0 t)] = (\mathbf{E}_{0x}\mathbf{e}_x + iE_{0y}\mathbf{e}_y)\exp[i(k_0 z - \omega_0 t)], \tag{1}$$

where $\mathbf{E}_0(\mathbf{r}, t)$ is the complex, slowly varying amplitude of the laser field and k_0 and ω_0 are the wave number and frequency, respectively. The plasma is embedded in a uniform axial magnetic field $\mathbf{b} = b\mathbf{e}_z$. We consider the ions to remain immobile and to form a neutralizing background, and consider only the electron response.

In the plasma, the equation of motion of an electron is

$$\frac{\mathrm{d}(\gamma_e m_e \mathbf{v}_e)}{\mathrm{d}t} = q_e\left[\mathbf{E} + \frac{1}{c}\mathbf{v}_e \times \tilde{\mathbf{B}}\right], \tag{2}$$

where m_e is the rest mass of an electron, γ_e is its relativistic factor, q_e is the electron charge, $\mathbf{v}_e$ is its velocity and $\tilde{\mathbf{B}}(= \mathbf{B} + \mathbf{b})$ is the total magnetic field.

Let us suppose that an inertial reference frame S' moves relative to the inertial reference frame S with velocity V along the z axis, where $V = nc$ and n is the refractive index. We use the primes for the corresponding quantities in the S' system. From the Maxwell's equations, we see that the total magnetic field $\tilde{\mathbf{B}}'$ in the S' system is equal to a constant vector and $\partial\mathbf{E}'/\partial t' = 4\pi N'|q_e|\mathbf{v}_e$, where N' is the electron number density and t' is the time. Assuming that there is only a constant magnetic field $\mathbf{b}'(= b'\mathbf{e}_{z'})$ in the S' system, Eq. (2) is reduced to

$$-m_e\frac{\mathrm{d}\mathbf{u}'_e}{\mathrm{d}t'} - \frac{|q_e|}{c^2}\mathbf{v}'_e \times \mathbf{b}' = \frac{|q_e|}{c}\mathbf{E}', \tag{3}$$

where $\mathbf{u}'_e = \gamma'_e\mathbf{v}'_e/c$ is the reduced velocity of electron. The second term on the left-hand side of Eq. (3) arises due to the interaction of electron quiver velocity with the constant magnetic field. Then the equation of motion of an electron in the S' system is

$$\frac{\mathrm{d}^2\mathbf{u}'_e}{\mathrm{d}t'^2} + \frac{\omega'^2_{pe}}{\gamma'_e}\mathbf{u}'_e + \frac{|q_e|}{m_e c^2}\frac{\mathrm{d}\mathbf{v}'_e}{\mathrm{d}t'} \times \mathbf{b}' = 0, \tag{4}$$

where $\omega'_{pe}(= \sqrt{4\pi N' q_e^2/m_e})$ is the electron plasma frequency in the S' system. We make an ansatz for the solution of Eq. (4) in the following form: $\mathbf{u}'_e = \mathbf{u}'_\perp e^{-i\omega' t'} + u'_{z'}\mathbf{e}_{z'}$, where $\mathbf{u}'_\perp$ and $u'_{z'}$ are slowly varying compared with the fluctuation at frequency ω' in frame S'. Substituting into Eq. (4), we obtain the dispersion equation in the S' system:

$$\omega'^2 = \frac{\omega'^2_{pe}}{\gamma'_e} + \frac{\omega'|q_e|b'}{m_e c\gamma'_e}. \tag{5}$$

The second term in Eq. (5) is due to the presence of the uniform magnetic field.

It is shown from Eq. (3) that

$$\mathbf{E}' = \mathbf{E}'_\perp e^{-i\omega' t'}, \tag{6}$$

where

$$\mathbf{E}'_\perp = \frac{i m_e c\omega'}{|q_e|}\mathbf{u}'_\perp - \frac{(\mathbf{u}'_e \times \mathbf{b}')_\perp}{\gamma'_e}.$$

Performing the Lorentz transformation and setting $\gamma_e^2 = 1 + u_\perp^2 + u_z^2$, Eq. (5) can be written as

$$\omega^2 = k^2c^2 + (1 + u_\perp^2)^{-\frac{1}{2}}\left(1 - \frac{v_0^2}{c^2}\right)^{\frac{1}{2}}\frac{\omega\omega_{pe}^2}{\omega - \omega_{Be}}, \tag{7}$$

where the cyclotron frequency $\omega_{Be}(= |q_e|b/\gamma_e m_e c)$ is supposed to be much lower than the frequency ω, $\omega_{pe}(= \sqrt{4\pi N|q_e|^2/m_e})$ is the electron plasma frequency, N is the number density, $u_z = \gamma_e v_0/c$, and v_0 is the electron flow velocity in the z-direction. Assuming $v_0 << c$ and taking Eq. (6) into account, we obtain the following nonlinear dispersion relation for the intense right-hand elliptically polarized laser field in magnetized plasma:

$$\omega^2 = k^2c^2 + \left[1 + \frac{q_e^2\mathbf{E}_\perp^2}{m_e^2c^2\omega^2}\left(1 + \frac{\omega_{Be}^2}{\omega^2}\right)^{-1}\right]^{-\frac{1}{2}}\frac{\omega\omega_{pe}^2}{\omega - \omega_{Be}}. \tag{8}$$

The second term on the right-hand side of Eq. (8) is due to relativistic effect and ponderomotive nonlinearities caused by the radiation and external magnetic fields. It may be noted that if the nonlinear term is switched off, Eq. (8) reduces to the well-known linear dispersion relation for the right-hand elliptically polarized electromagnetic wave propagating in plasma. The transformed electric field in the S system is

$$\mathbf{E} = \mathbf{E}_\perp e^{-i(\omega t - kz)}. \tag{9}$$

And Eq. (9) becomes $\mathbf{E} = \mathbf{\Psi}e^{-i(\omega_0 t - k_0 z)}$, where $\mathbf{\Psi}(= \mathbf{E}_\perp e^{-i[(\omega-\omega_0)t-(k-k_0)z]})$ is the complex, slowly varying amplitude of the laser field and it is governed by the nonlinear evolution equation.

3. Nonlinear evolution equation for the envelope of the laser electric field

The envelope of the transverse pump wave is represented by $\mathbf{\Psi}(\mathbf{r}, t) = \mathbf{a}(\mathbf{r}, t)\exp[i\theta(\mathbf{r}, t)]$, where $\theta(\mathbf{r}, t) = k_0 z - \omega_0 t + \phi(\mathbf{r}, t)$. In the short-wavelength approximation, one can find $-\partial\theta/\partial t \equiv \omega = \omega_0 - \partial\phi/\partial t$ and $\nabla\theta \equiv \mathbf{k} = \mathbf{k}_0 + \nabla\phi$, where $\mathbf{k}_0(= k_0\mathbf{e}_z)$ is the wave vector. We expand the dispersion relation $\omega(k)$ in a Taylor series about k_0 and retain terms up to third order:

$$\omega(k) = \omega(k_0) + \left(\frac{\partial\omega}{\partial k}\right)\bigg|_{k=k_0}(k - k_0) + \frac{1}{2}\left(\frac{\partial^2\omega}{\partial k^2}\right)\bigg|_{k=k_0}(k - k_0)^2. \tag{10}$$