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Creation of cluster state of four ions in ion-trap system by individual addressing

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ABSTRACT

The cluster state is a special, highly entangled quantum state that forms the universal resource, on which measurement-based quantum computation can be performed. In this study, a new scheme is presented for creating four-ions cluster state in ion-trap system. This scheme is based on resonant sideband excitation in which the population is transferred to target states by precise control of pulse area. Meanwhile, the scheme is consist of combination of elementary stages belonging to a universal set whereby cluster state has been created in five interaction stages by individually addressed ions with red- or blue-sideband resonance laser pulses. The paper studies the population transfer of the system by numerical solutions of the master equation, considering the effect of decoherence channels due to dissipation in the phonon modes. The presented scheme does not require control of the turn-off ratio and time delay among pulses.

1. Introduction

Entanglement, especially multi-particle entanglement is a key ingredient in quantum information processing [1]. To date, in the case of tripartite entanglement, there are two inequivalent classes of tripartite entangled states, the GHZ class [2] and the W class [3]. Additionally, multi-particle entanglement, namely Cluster State, was first introduced by Briegel and Raussendorf [4]. The cluster states have high persistence of entanglement and can be regarded as an entanglement resource for the GHZ states but are more immune to decoherence than them [3]. The cluster states have extensive application in a way that this special highly entangled states are essential element in one-way quantum computing [5,6] and implementing quantum communication [7]. Because of both conceptual and practical importance, preparation of cluster states has attracted much attention. To date, various schemes for generating cluster states in different physical systems have been proposed, such as; scheme for preparing a cluster state by many superconducting quantum-interference device qubits in cavities [8], a scheme for generating a four-photon polarization-entangled cluster state by using linear optical elements [9] and scheme for producing a four-qubit cluster state via ion trap systems [10,11]. Among these, trapped ions are the most promising physical system for implementing quantum computing due to their long coherence times and individual addressing [12] in which the information is stored in internal states and is transferred via external (motional) states of the ions [13].

In this paper, a novel scheme is proposed for creation of cluster state of four identical two-state ions in which the ions are confined in a linear potential in ion-trap system. The ions are addressed individually in five interaction stages and the blue- or red-sideband resonance laser pulses tuned to the ions in an innovative sequence. The presented scheme is based on the linkage pattern presented by Ivanov et al. [11]. Indeed, Ivanov et al. [11] have proposed a scheme for creating four ion cluster state in which each ion interacts with pair of red- and blue-sideband laser pulses simultaneous, while we implemented specific sequence of red- or blue-sideband coupling to the specific ions during the stages. Moreover in Ref. [11], the blue and red couplings for each ion have been assumed equal and have the same time dependence. While, in this study blue and red coupling for each ion are imposed to be pulse shaped possibly with different time dependence and magnitude.

In this paper, in order to create the superposition states, coherence resonance excitation has been implemented. Resonant pulses of special pulse areas implement in a variety of fields in quantum physics including quantum information processing [14,15], nuclear magnetic resonance [16] and coherent atomic excitation [17]. This

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technique provides great facility in establishing a full control over the quantum system especially in two-state systems. Furthermore, the particular use of resonance pulses, known as π-pulse, is used broadly in quantum information in order to achieve important transformations including complete coherent population transfer, complete population return and Hadamard transformation [1]. In this study the generalized resonant π-pulse technique, which has been studied in Ref. [18] for creation of quNit states, is implemented for creation of superposition states. It is shown in the manuscript. By precise controlling of both pulse duration and peak Rabi frequency, complete population is transferred to the desired target states coherently whereby the implemented laser pulses are tuned without any need for controlling of the turn-off ratio and time delay among them, additionally transitions are in complete resonance.

As it was mentioned previously, in this paper the cluster state of the four trapped ions is created in five interaction stages. The numerical solution of population transfer for each stage has been calculated in the paper and the population transfer of each stage has been illustrated. The Morris-Shore (MS) transformation has been used to decomposition of degenerate states into set of independent states [19,20]. This paper has been organized as follow. In Section 2, the model for creating of corresponding highly entangled cluster state is presented. In Section 3, the population transfer mechanism is explained in five interaction stages. In Sections 3.1–3.3, the population is distributed to the neighbor states via appropriate adjustment of pulse areas. In Sections 3.4 and 3.5, the population is transferred from populated neighbor states to the desired entangled cluster state. In Section 4, decoherence channels caused by dissipation in the phonon modes has been discussed. Finally, Section 5 provides conclusion for paper.

2. The model to create cluster state of four ions

The cluster state forms a class of multipartite entangled quantum state with useful properties. This entangled state has more persistence entanglement comparison to GHZ state and is universal resource in one way quantum computing. In this study, in order to create a four ions cluster state, the linkage pattern presented by Ivanov et al. [11] is utilized. The linkage pattern illustrated in Fig. 1, represents the linkage pattern of chain of four identical two-state ions in which the red and blue color linkages represents ions interaction with laser fields by frequencies tuned near the red- and blue-sideband transitions respectively. Moreover the laser-atom detuning δ, is taken $\delta = 0$. We implemented an innovative and specific sequence of resonance laser pulses, with red- or blue-sidebands for creating cluster state of a chain of four ions in ion-trap system. The cluster state is achieved in five interaction stages in which during the each stage the population is transferred to different states with the purpose of creation of cluster state.

We have implemented Rabi frequency laser pulses ($\Omega_j(t) = |\Omega_j(t)|e^{i\phi_j}$, $j = 1, 2, \ldots, 8$) to couple ground state $|g\rangle$ and excited state $|e\rangle$ where laser pulses are regarded as real valued with phase ϕ_j which is taken ($\phi_j = 0$). In Fig. 1, $\Omega_1(t)$, $\Omega_2(t)$, $\Omega_3(t)$ and $\Omega_4(t)$, are red-sideband resonances and $\Omega_5(t)$, $\Omega_6(t)$, $\Omega_7(t)$ and $\Omega_8(t)$, are blue-sideband resonance laser pulses that are tuned to the first, second, third and forth ion, respectively. In this way, by tuning laser pulses to each ions, eight two-state couplings appear. In the Lamb-Dick regime and rotating-wave approximation, the interaction Hamiltonian describing the red-and blue-sideband transition respectively are as follow [12,13];

$$H_r = i\eta \sum_{j=1}^{4} \Omega_j (e^{-i\phi_j} a^\dagger |g\rangle_j \langle e| + e^{i\phi_j} a |e\rangle_j \langle g|), \tag{1a}$$

$$H_b = i\eta \sum_{j=5}^{8} \Omega_j (e^{i\phi_j} a^\dagger |e\rangle_j \langle g| - e^{-i\phi_j} a |g\rangle_j \langle e|), \tag{1b}$$

In Eqs. (1a) and (1b), Ω_j is Rabi frequency of laser fields, ϕ_j is phase of the jth laser pulses which is assumed $\phi_j = 0$, $a^\dagger(a)$ denotes creation (annihilation) operator of the vibrational mode (phonons), $(|e\rangle_j\langle g| = \sigma_j^+)$ is the atomic raising and $(|g\rangle_j\langle e| = \sigma_j^-)$ is the atomic lowering operators. In the following the states of ion1-ion2-ion3-ion4-vibrational mode system considered as $|i_1, i_2, i_3, i_4, n\rangle$ where $\{i_1, i_2, i_3, i_4 = g, e$ & $g = 0, e = 1\}$ and n $\{n = 0, 1, 2\}$ is the number of vibrational phonons in respective mode. Regarding linkage pattern in Fig. 1, the subspace $\mathcal{S}$ generated by the states $\{|1000\rangle|0\rangle, |0100\rangle|0\rangle, |0010\rangle|0\rangle, |0001\rangle|0\rangle, |1110\rangle|0\rangle, |1101\rangle|0\rangle, |1011\rangle|0\rangle, |0111\rangle|0\rangle, |0000\rangle|1\rangle, |1100\rangle|1\rangle, |1010\rangle|1\rangle, |1001\rangle|1\rangle, |0110\rangle|1\rangle, |0101\rangle|1\rangle, |0011\rangle|1\rangle, |1111\rangle|1\rangle, |1000\rangle|2\rangle, |0100\rangle|2\rangle, |0010\rangle|2\rangle, |0001\rangle|2\rangle, |1110\rangle|2\rangle, |1101\rangle|2\rangle, |1011\rangle|2\rangle, |0111\rangle|2\rangle\}$ are decoupled under H_b and H_r from the rest of the Hilbert space of the system. The initial state of the system assumed $|0000\rangle|1\rangle$, so that the effective Hamiltonian of system in the subspace $\mathcal{S}$ is as below:

$$H^{\text{eff}}(t) = \begin{pmatrix} 0 & V^\dagger_{n,n+1} & 0 \\ V_{n,n+1} & 0 & V^\dagger_{n-1,n} \\ 0 & V_{n-1,n} & 0 \end{pmatrix}. \tag{2}$$

In Eq. (2) the matrix elements, $V_{n,n+1}$ and $V_{n-1,n}$, are 8×8 interaction matrices containing the coupling of middle set with upper set and middle set with lower set respectively (see Fig. 1). The matrix element $H^{\text{eff}}(2, 2)$ is a 8×8 matrix which is equal to zero and reflects that the middle states have no interaction with each others. The matrix element, $H^{\text{eff}}(1, 1)$ and H^{eff} (3, 3), are 8×8 matrices show detuning has been taken zero ($\delta = 0$). The matrix elements, $H^{\text{eff}}(3,1)$ and $H^{\text{eff}}(1,3)$ are also 8×8 matrices indicating that lower and upper states have not direct coupling. The matrices elements, $V_{n,n+1}$ and $V_{n-1,n}$ are;

$$V_{n,n+1}(t) = \begin{pmatrix} \Omega_1(t) & \Omega_2(t) & \Omega_3(t) & \Omega_4(t) & 0 & 0 & 0 & 0 \\ \Omega_6(t) & \Omega_5(t) & 0 & 0 & \Omega_3(t) & \Omega_4(t) & 0 & 0 \\ \Omega_7(t) & 0 & \Omega_5(t) & 0 & \Omega_2(t) & 0 & \Omega_4(t) & 0 \\ \Omega_8(t) & 0 & 0 & \Omega_5(t) & 0 & \Omega_2(t) & \Omega_3(t) & 0 \\ 0 & \Omega_7(t) & \Omega_6(t) & 0 & \Omega_1(t) & 0 & 0 & \Omega_4(t) \\ 0 & \Omega_8(t) & 0 & \Omega_6(t) & 0 & \Omega_1(t) & 0 & \Omega_3(t) \\ 0 & 0 & \Omega_8(t) & \Omega_7(t) & 0 & 0 & \Omega_1(t) & \Omega_2(t) \\ 0 & 0 & 0 & 0 & \Omega_8(t) & \Omega_7(t) & \Omega_6(t) & \Omega_5(t) \end{pmatrix}, \tag{3}$$