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The quantum correlation of anisotropic Heisenberg spin chain under the control of inhomogeneous magnetic field

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ABSTRACT

The quantum correlation (quantum entanglement and quantum discord)'s dynamical behavior characteristics of Heisenberg *XXZ* spin chain with Dzyaloshinskii–Moriya (DM) interaction heterogeneous magnetic field that manipulated by sinusoidal wave are investigated in this paper. The results indicate that quantum correlation of anisotropic Heisenberg *XXZ* spin chain can be regulated effectively by magnetic field intensity B and magnetic field uniformity $\cos\theta$ of the external magnetic field. Under the effects of DM interaction in qubits, the quantum correlation's dynamics evolution process appears sudden death and birth. But DM interaction has a critical value D_{CZ} which is connected with other quantum correlation versus parameters. Only when $D_z \geq D_{CZ}$, sudden death and birth can be obviously observed under the rest given parameters.

1. Introduction

Among the vast quantum systems, solid state quantum system plays an important role when people study quantum entanglement [1] owing to its abundant energy levels, boundary conditions, easy accessibility and integration. In order to realize quantum information processing, quantum entanglement needs to be manipulated. So entanglement properties and entanglement's time-evolution have been extensively studied by several researchers. Many valuable fruits have been achieved [2–4]. The study results show that beside the interactional anisotropic factors in the spin chain, the external environment [5,6] (such as temperature [7] and magnetic field [8]) also has significant influence on the entanglement. Therefore entanglement resources is obtained efficiently through regulating anisotropy [9], temperature [10] and the intensity or direction of magnetic field and is applied to quantum computing [11] and quantum teleportation. For example, it can be used to realize basic quantum logic gate, quantum manipulation, quantum coding, quantum cloning and so on.

However, it is challengeable for one to exploit quantum entanglement since the decoherence effect caused by environment (including measuring instruments) will lead to disentanglement even entanglement death [12]. On one hand, people have researched deeper on how to control death and birth of entanglement and have regulated quantum entanglement in order to implement quantum information processing effectively. On the other hand, they have devoted to find new quantum information resources. Recently, it has been shown that there are also quantum tasks that displaying the quantum advantages without entanglement, for example, the deterministic quantum computation with one qubit [13]. This shows that entanglement does not account for all the nonclassical properties of quantum states. At present, the quantum discord introduced by Olliver and Zurek is used for measuring the quantum correlations [14].

By counting DM interaction into consideration, the thermal quantum entanglement of Heisenberg *XXZ* spin chain has been studied in Ref. [15]. The results show that DM interaction is beneficial to raise the critical temperature. Ref. [16] has investigated thermal quantum entanglement and quantum teleportation of Heisenberg *XXZ* spin chain with DM interaction. It has been shown that DM interaction is an efficacious parameter to manipulate quantum entanglement. However, studies on quantum correlation of anisotropic Heisenberg spin chain under nonuniform external magnetic field is not wide enough at present especially the quantum correlation in Heisenberg spin chain with DM interaction under inhomogeneous external magnetic field [17]. It is verified theoretically and experimentally that heterogeneity of system intrinsic coupling parameter strengthens quantum entanglement. Heterogeneity of intrinsic coupling is hard to manipulate. So that, more attention should be paid to easy-manipulative magnetic field parameters' (magnetic field intensity and heterogeneity) affects on quantum correlation. In this paper, we investigate quantum

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correlation in Heisenberg *XXZ* spin chain with DM interaction heterogeneous magnetic field modulated by a sinusoidal wave.

2. Model

We consider an anisotropic Heisenberg *XXZ* spin chain model with DM interaction under sinusoidal impulse magnetic field. The Hamiltonian of system is given as [18]:

$$\hat{H} = \sum_{j=1}^{N} J(\hat{\sigma}_j^x\hat{\sigma}_{j+1}^x + \hat{\sigma}_j^y\hat{\sigma}_{j+1}^y) + J_z\hat{\sigma}_j^z\hat{\sigma}_{j+1}^z + B(\hat{\sigma}_{2j-1}^z + \hat{\sigma}_{2j}^z\cos\theta) + D_z(\hat{\sigma}_j^x\hat{\sigma}_{j+1}^y - \hat{\sigma}_j^y\hat{\sigma}_{j+1}^x), \quad (1)$$

where $\hat{\sigma}_j^\alpha$ $(\alpha = x, y, z; \quad n = 1, 2, \ldots, N)$ is the Pauli matrix for qubit j. The first and second terms are the coupling ones. The third term is Zeeman term. The last term is DM interaction. J is the coupling coefficient in x and y directions and J_z is the coupling coefficient in z direction. We only consider the antiferromagnetic case ($J > 0$, $J_z > 0$). This model is reduced to the isotropic XX model when $J_z = 0$ and to the isotropic XXX model when $J_z = J$. In this paper, we focus on the two-spin chain system. Then, $B \geq 0$ is restricted and the magnetic fields on the two spins have been so parameterized that $\cos\theta$ controls the degree of nonuniformity. The model of heterogeneous magnetic field described in this paper is universal since the external magnetic field manipulated by the sinusoidal wave is introduced.

The density matrix of the system can be written as

$$\rho(t) = U(t)|\psi(0)\rangle\langle\psi(0)|U^+(t). \quad (2)$$

$U(t)$, time-evolution operator of whole system, can be written as

$$U(t) = \exp\left(-\frac{i}{\hbar}\hat{H}t\right) = \sum_j \exp\left(-\frac{i}{\hbar}E_j t\right)|\varphi_j\rangle\langle\varphi_j|. \quad (3)$$

E_j is eigenvalues. $|\varphi_j\rangle$ is eigenvectors. The analytical eigenvalues can be achieved as follows

$$E_1 = J_z - x,\ E_2 = J_z + x,\ E_3 = -J_z - \Omega,\ E_4 = -J_z + \Omega,$$

while

$$x = B(1 + \cos\theta), \quad y = B(1 - \cos\theta), \quad \Omega = \sqrt{4(J^2 + D_z^2) + y^2}.$$

The adiabatic eigenvectors are given by:

$$|\psi_1\rangle = |00\rangle,$$

$$|\psi_2\rangle = |11\rangle,$$

$$|\psi_3\rangle = \frac{1}{\sqrt{4(J^2 + D_z^2) + (y - \Omega)^2}}\left[\frac{(y - \Omega)(J + iD_z)}{\sqrt{J^2 + D_z^2}}|01\rangle + 2\sqrt{J^2 + D_z^2}|10\rangle\right],$$

$$|\psi_4\rangle = \frac{1}{\sqrt{4(J^2 + D_z^2) + (y + \Omega)^2}}\left[\frac{(y + \Omega)(J + iD_z)}{\sqrt{J^2 + D_z^2}}|01\rangle + 2\sqrt{J^2 + D_z^2}|10\rangle\right].$$

Based on eigenvalues and eigenvectors above, time-evolution operator of whole system in the standard basis $|00\rangle, |01\rangle, |10\rangle, |11\rangle$ can be written as

$$U(t) = \begin{pmatrix} e^{-iE_1 t/\hbar} & 0 & 0 & 0 \\ 0 & e^{iJ_z t/\hbar}\left(\cos\alpha + i\frac{y}{\Omega}\sin\alpha\right) & -i\frac{2(J - iD_z)}{\Omega}e^{iJ_z t/\hbar}\sin\alpha & 0 \\ 0 & -i\frac{2(J + iD_z)}{\Omega}e^{iJ_z t/\hbar}\sin\alpha & e^{iJ_z t/\hbar}\left(\cos\alpha - i\frac{y}{\Omega}\sin\alpha\right) & 0 \\ 0 & 0 & 0 & e^{-iE_2 t/\hbar} \end{pmatrix}, \quad (4)$$

$\alpha = \Omega t/\hbar$.

Without losing the generality, in the following discussion, we consider two classes of initial sates,

$$\rho_\psi(r, \theta) = r|\psi(\varphi)\rangle\langle\psi(\varphi)| + \frac{1 - r}{4}I_4, \quad (5)$$

where r is the purity of the initial states which ranges from 0 for maximally mixed states to 1 for pure states, I_4 is the 4×4 identity matrix. $|\psi(\varphi)\rangle = \cos\varphi|01\rangle + \sin\varphi|10\rangle$ is the Bell-like pure state and the parameter θ is sometimes called the degree of entanglement.

$$\rho_{11}(t) = \frac{1 - r}{4}, \quad (6)$$

$$\rho_{22}(t) = \left(r\sin^2\varphi + \frac{1 - r}{4}\right)\left(\cos^2\alpha + \frac{y^2}{\Omega^2}\sin^2\alpha\right) + (4r\cos^2\varphi - r + 1)\frac{(J^2 + D^2)}{\Omega^2}\sin^2\alpha - \frac{r\sin 2\varphi}{\Omega}\left(\frac{2Jy}{\Omega}\sin^2\alpha + D\sin 2\alpha\right), \quad (7)$$

$$\rho_{23}(t) = \left(\cos\alpha + i\frac{y}{\Omega}\sin\alpha\right)\left[\frac{r}{2}\left(\cos\alpha + i\frac{y}{\Omega}\sin\alpha\right)\sin 2\varphi - i\frac{2r(J - iD)\cos 2\varphi}{\Omega}\sin\alpha\right] + \frac{2r(J - iD)^2\sin 2\varphi}{\Omega^2}\sin^2\alpha \quad (8)$$

$$\rho_{32}(t) = \left(\cos\alpha - i\frac{y}{\Omega}\sin\alpha\right)\left[\frac{r}{2}\left(\cos\alpha - i\frac{y}{\Omega}\sin\alpha\right)\sin 2\varphi + i\frac{2r(J + iD)\cos 2\varphi}{\Omega}\sin\alpha\right] + \frac{2r(J + iD)^2\sin 2\varphi}{\Omega^2}\sin^2\alpha, \quad (9)$$

$$\rho_{33}(t) = \left(r\cos^2\varphi + \frac{1 - r}{4}\right)\left(\cos^2\alpha + \frac{y^2}{\Omega^2}\sin^2\alpha\right) + (4r\sin^2\varphi - r + 1)\frac{(J^2 + D^2)}{\Omega^2}\sin^2\alpha + \frac{r\sin 2\varphi}{\Omega}\left(\frac{2Jy}{\Omega}\sin^2\alpha + D\sin 2\alpha\right) \quad (10)$$

$$\rho_{44}(t) = \frac{1 - r}{4}, \quad (11)$$

Other elements of matrix are zero.

3. Measure of quantum correlation

We adopt the concurrence entanglement defined by Wootters to measure the system entanglement [19]. Under X-state [20],

$$C(t) = 2\max\{0, \quad |\rho_{14}(t)| - \sqrt{\rho_{22}(t)\rho_{33}(t)}, \quad |\rho_{23}(t)| - \sqrt{\rho_{11}(t)\rho_{44}(t)}\}. \quad (12)$$