Optik 125 (2014) 6539–6542

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Analysis of a right-angle conical reflector resonator by the transfer matrix method

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ARTICLE INFO

Article history:
Received 22 November 2013
Accepted 20 June 2014

Keywords:
Right angle conical reflector resonator
The transfer matrix method
Transverse mode

ABSTRACT

A resonator with a right-angle conical reflector has been proposed to produce high-power CO_2 laser beams. To analyze eigenfields of the right-angle conical reflector resonator, this paper adopts and demonstrates the transfer matrix method. In this paper, the mode-fields and corresponding losses are described as eigenvectors and eigenvalues of a transfer matrix according to the self-reproducing principle of laser field. By solving the transfer matrix for eigenvectors and eigenvalues, we obtain field distributions and losses of the dominant eigenmodes. The calculation results reveal that the right-angle conical reflector resonator could be used for a high-power CO_2 laser to achieve low-order modes. However, the beam quality is reduced due to the residual blind-hole, which is in accord with the experimental result.

1. Introduction

Based on the resonators with a retroreflecting roof mirror and a retroreflecting corner cube mirror [1–3], a right-angle cone mirror cavity for a TEA CO_2 laser has been proposed and experimentally studied [4,5], it is clear from the experimental results that the right-angle cone mirror cavity improves the capability against misalignment and the distribution of output beams in near field are uniform although the pulse output power of the TEA CO_2 laser is similar to that of the laser by use of a plano-concave resonator. As a consequence, in this paper, to analyze eigenfields of the resonator with a 90° conical reflector accurately and conveniently, a transfer matrix algorithm which combines the self-reproducing principle of optical field with the diffraction integral in the form of ray matrix are adopted.

As has been well known, the Huygens–Fresnel diffraction integral in the form of ray matrix can be applied in the optical beam propagation and transformation of an ABCD optics system [6]. Consequently, in this paper diffraction integral equations in the form of ray matrices are transformed into finite-sum matrix equations [7,8]. Following the laser self-reproducing condition, i.e., a state is reached in which the relative field distribution does not vary from transit to transit and the amplitude of the field decays at an exponential rate, we describe mode-fields and their losses of the 90° conical reflector resonator as eigenvectors and eigenvalues of a transfer matrix. Finally, field distributions and losses of the 90° conical reflector resonator are obtained by use of the matrix numeration. As we will demonstrate, the whole analysis method proposed by this paper can be adopted in estimating the modes of output beams.

The rest of this paper is organized as follows: Section 2 describes first the configuration of a 90° conical reflector resonator and writes out the corresponding one-way diffraction integral equations. In Section 3, according to the laser self-producing condition, the Huygens–Fresnel diffraction integral equations in the form of ray matrices are converted into the matrix equations. Section 4 calculates and analyzes eigenfields and their losses of the 90° conical reflector resonator, while our conclusions are drawn in Section 5.

2. The configuration of the resonator and its integral equations

As described above, a resonator with a 90° conical reflector has been proposed to output high-power CO_2 laser beams, not needing complex configuration. Here, we cite the configuration adopted by Hongqi Li and Zuhai Cheng [4], as shown in Fig. 1.

The resonator consists of a 90° conical reflector M_1, and a flat mirror M_2, which is placed at a distance L from the bottom of the 90° conical reflector and serves as the output coupler. The radii of both the axicon M_1 and the output coupler M_2 are a. The dashed ellipse represents the blind-hole, namely, the top of the conical reflector is irreflexible.

The diffraction integral derived by Baues and Collins ignores diffraction losses induced by diffraction-limited optical elements

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http://dx.doi.org/10.1016/j.ijleo.2014.08.045
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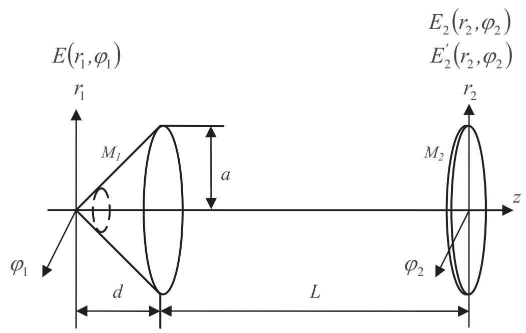

Fig. 1. Scheme of the right-angle conical reflector resonator.

inside an optics system. Accordingly, in order to calculate accurately eigenmodes and their losses of the resonator, the round-trip laser transmission should be disintegrated into two one-way transmissions; otherwise the diffraction loss caused by one mirror is ignored if the round-trip ray matrix is directly used.

As denoted in Fig. 1, after reflected by M_2 and propagating from M_2 to M_1, the one-way ray can be described by the following matrix:

$$\mathbf{T}_1=\begin{bmatrix}A_1 & B_1\\ C_1 & D_1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & L\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & L\\ 0 & 1\end{bmatrix}. \quad (1)$$

Similarly, the ray matrix for one-way ray reflected by M_1 and propagating from M_1 to M_2 is:

$$\mathbf{T}_2=\begin{bmatrix}A_2 & B_2\\ C_2 & D_2\end{bmatrix}=\begin{bmatrix}1 & L\\ 0_1 & 1\end{bmatrix}\begin{bmatrix}-1 & -2d\\ 0 & -1\end{bmatrix}=\begin{bmatrix}-1 & -2d-L\\ 0 & -1\end{bmatrix}. \quad (2)$$

As pointed by Collins, if an incident optical field propagates through an axisymmetric optics system described by an $ABCD$ ray matrix, the Huygens–Fresnel diffraction integral can be expressed as [6].

$$E_2(r_2,\varphi_2)=-\frac{ik\exp(ikL)}{2\pi B}\iint_{S_1}E_1(r_1,\varphi_1)$$
$$\exp\left\{\frac{ik}{2B}\left[Ar_1^2+Dr_2^2-2r_1r_2\cos(\varphi_1-\varphi_2)\right]\right\}r_1dr_1d\varphi_1, \quad (3)$$

where r_1 and φ_1 are the cylindrical coordinates for the incident plane, $E_1(r_1, \varphi_1)$ is the incident optical field, S_1 represents the incident plane, r_2 and φ_2 are the cylindrical coordinates for the diffraction plane, $E_2(r_2, \varphi_2)$ is the diffraction optical field, λ is the light wavelength, k is termed the wave number given by $k=2\pi/\lambda$, i represents the imaginary unit $\sqrt{-1}$, and A, B, and D are the elements of the ray matrix.

Given an original field $E_2(r_2, \varphi_2)$ just before the plane mirror M_2, from Eqs. (1) and (3), the diffracted field across the interface 1 is:

$$E_1(r_1,\varphi_1)=-\frac{ik\exp(ikL)}{2\pi B_1}\iint_{S_2}E_2(r_2,\varphi_2)$$
$$\exp\left\{\frac{ik}{2B_1}\left[A_1r_2^2+D_1r_1^2-2r_1r_2\cos(\varphi_1-\varphi_2)\right]\right\}r_2dr_2d\varphi_2. \quad (4)$$

By using Eqs. (2) and (3), the diffraction field across the interface 2 can be written as

$$E_2'(r_2,\varphi_2)=-\frac{ik\exp(ikL)}{2\pi B_2}\iint_{S_1}E_1(r_1,\varphi_1)$$
$$\exp\left\{\frac{ik}{2B_2}\left[A_2r_1^2+D_2r_2^2-2r_1r_2\cos(\varphi_1-\varphi_2)\right]\right\}r_1dr_1d\varphi_1. \quad (5)$$

Making a separation of variables for light fields along r and φ direction, namely:

$$E_1(r_1,\varphi_1)=E_1(r_1)\exp(in\varphi_1),\ E_2(r_2,\varphi_2)=E_2(r_2)\exp(in\varphi_2),$$
$$E_2'(r_2,\varphi_2)=E_2'(r_2)\exp(in\varphi_2), \quad (6)$$

where n is an integer, and substituting Eq. (6) into Eqs. (4) and (5), respectively, we write the diffraction equations along r direction as

$$E_1(r_1)=\frac{(-i)^{n+1}k\exp(ikL)}{B_1}\int_0^a E_2(r_2)J_n\left(\frac{kr_1r_2}{B_1}\right)$$
$$\exp\left[\frac{ik}{2B_1}\left(A_1r_2^2+D_1r_1^2\right)\right]r_2dr_2, \quad (7)$$

$$E_2'(r_2)=\frac{(-i)^{n+1}k\exp(ikL)}{B_2}\int_0^a E_1(r_1)J_n\left(\frac{kr_1r_2}{B_2}\right)$$
$$\exp\left[\frac{ik}{2B_2}\left(A_2r_1^2+D_2r_2^2\right)\right]r_1dr_1, \quad (8)$$

where J_n is the nth-order Bessel function.

As have done above, the corresponding diffraction integral equations for the 90° conical reflector resonator have been obtained which are in relation to the ray matrices given by Eqs. (1) and (2). Evidently, self-reproducing eigenmodes cannot be analytically derived from Eqs. (7) and (8). Hence, in the following section, we will induce the matrix equations of the 90° conical reflector resonator.

3. The matrix equations for eigenmodes of the 90° conical reflector resonator

From the above discussion, it is readily distinct that eigenfields of the resonator with a 90° conical reflector are determined by diffraction integral equations along r direction. Therefore, the eigenmode matrix equation of the resonator with a 90° conical reflector can be derived according to the laser self-reproducing condition as long as Eqs. (7) and (8) are converted into matrix equations.

Dividing two mirrors M_1 and M_2 into M units (circular rings) which have equivalent lengths along r direction, respectively, and using Eqs. (7) and (8), we acquire discrete distributions of the diffracted fields along r direction as follows:

$$E_1(r_1)_m=\sum_{n=1}^{M}X_{mn}E_2(r_2)_n, \quad (9)$$

$$E_2'(r_2)_m=\sum_{n=1}^{M}Y_{mn}E_1(r_1)_n, \quad (10)$$

where the elements of one-way matrices $\mathbf{X}$ and $\mathbf{Y}$ are given by

$$X_{mn}=\frac{(-i)^{nn+1}kna^2\exp(ikL)}{B_1M^2}J_{nn}\left(\frac{kmna^2}{B_1M^2}\right)$$
$$\exp\left[\frac{ia^2\pi}{B_1\lambda M^2}\left(A_1n^2+D_1m^2\right)\right], \quad (11)$$

$$Y_{mn}=\frac{(-i)^{nn+1}kna^2\exp(ikL)}{B_2M^2}J_{nn}\left(\frac{kmna^2}{B_2M^2}\right)$$
$$\exp\left[\frac{ia^2\pi}{B_2\lambda M^2}\left(A_2n^2+D_2m^2\right)\right], \quad (12)$$