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Adaptive impulsive synchronization for a class of fractional-order chaotic and hyperchaotic systems

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ABSTRACT

In this paper, the adaptive impulsive synchronization for a class of fractional-order chaotic and hyperchaotic systems with unknown Lipschitz constant is investigated. Firstly, based on the adaptive control theory and the impulsive differential equations theory, the impulsive controller, the adaptive controller and the parametric update law are designed, respectively. Secondly, by constructing the suitable response system, the original fractional-order error system can be converted into the integral-order one. Finally, the new sufficient criterion is derived to guarantee the asymptotical stability of synchronization error system by the Lyapunov stability theory and the generalized Barbalat's lemma. In addition, numerical simulations demonstrate the effectiveness and feasibility of the proposed adaptive impulsive control method.

1. Introduction

Fractional derivative has its inception in an exchange of letters between L'Hospital and Leibniz in 1695. It is observed that the description of some systems is more accurate when the fractional derivative is used [1–4]. Compared to classical integral-order models, the fractional derivative provides an excellent instrument to describe the memory and hereditary properties of various materials and processes. So research on fractional-order systems has a more universal meaning. Nowadays, many fractional-order differential systems behave chaotically [5–8]. Chaotic synchronization in fractional-order chaotic and hyperchaotic systems is becoming the research hotspot of nonlinear science [9–11], due to its wide applications in secure communication and control processing.

In recent years, a variety of approaches have been proposed for the synchronization of fractional-order chaotic systems such as active control method [12,13], sliding mode control method [14,15], adaptive control method [16,17], impulsive control method [18–20] and so on. As impulsive control allows the stabilization and synchronization of chaotic systems using only small control impulsive, it has been widely used to stabilize and synchronize chaotic systems [21–23]. In [18], a novel impulsive control method based on comparison system was reported to achieve complete synchronization of a class of fractional-order chaotic systems. An impulsive synchronization scheme for a class of fractional-order hyperchaotic systems was proposed in [19]. A new synchronization criterion of fractional-order chaotic systems was proposed based on the stability theory of impulsive fractional-order systems in [20]. However, there is little related results reported on adaptive impulsive synchronization of fractional-order chaotic and hyperchaotic systems. Research in this area should be challenging. In [24], by the generalized Barbalat's lemma and the Lyapunov stability theory, Zhang et al. investigated the adaptive impulsive synchronization for a class of non-autonomous integral-order chaotic systems with unknown Lipschitz constant. In [25], Li et al. discussed the issue of adaptive impulsive synchronization and parameter identification for a class of integral-order chaotic and hyperchaotic systems. In this paper, we will discuss the adaptive impulsive synchronization for a class of fractional-order chaotic and hyperchaotic systems with unknown Lipschitz constant, based on Ref. [18,24,25]. Numerical simulations are presented to verify the effectiveness of this approach.

The rest of the paper is organized as follows. In Section 2, some preliminaries of fractional derivative are briefly introduced. Adaptive impulsive synchronization method of fractional-order chaotic systems is presented in Section 3. In Section 4, the proposed method is applied to fractional-order chaotic and hyperchaotic Chen systems with unknown Lipschitz constant. Simulation results are shown. Finally, conclusions are addressed in Section 5.

2. Preliminaries of fractional derivative

At present, there are several definitions of fractional-order differential operator, such as Grünwald–Letnikov (GL) definition,

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Riemann–Liouville (RL) definition, Caputo definition, and Jumarie definition. Among them, the method defined by GL is the most direct numerical one to solve the fraction-order system. Now we give GL definition as follows [26,27]:

$$ {}_aD_t^{\alpha}f(t)=\lim_{t\to 0}h^{-\alpha}\sum_{j=0}^{(t-a)/h}(-1)^j\begin{pmatrix} q \\ j \end{pmatrix}f(t-jh) \tag{1} $$

Eq. (1) can be reduced to

$$ {}_aD_t^{\alpha}y(m)=h^{-\alpha}\sum_{j=0}^{m}\omega_j^{(\alpha)}y_{m-j}, \tag{2} $$

where

$$ \begin{pmatrix} \alpha \\ j \end{pmatrix}=\frac{\alpha(\alpha-1)\cdots(\alpha-j+1)}{j!}, $$

$$ \omega_j^{(\alpha)}=(-1)^j\begin{pmatrix} \alpha \\ j \end{pmatrix}, \quad j=0,1,2,\cdots $$

h is the time step. The above approximation is mainly used in this paper.

3. Description of adaptive impulsive synchronization

Consider a class of fractional-order system

$$ D^{\alpha}\boldsymbol{x}(t)=A\boldsymbol{x}(t)+\phi(\boldsymbol{x}(t)), \tag{3} $$

where $0<\alpha<1$, $\boldsymbol{x}\in\Re^n$ represents the state vectors of the system, $A\in\Re^{n\times n}$, $\phi:\Re^n\to\Re^n$ is nonlinear vector function. System (3) is regarded as the drive system.

Assumption 3.1. For any $\boldsymbol{x},\boldsymbol{y}\in\Omega\subseteq\Re^n$, $\exists L>0$ such that

$$ \|\phi(\boldsymbol{y}(t))-\phi(\boldsymbol{x}(t))\|\le L\cdot\|\boldsymbol{y}(t)-\boldsymbol{x}(t)\|. \tag{4} $$

Remark 3.1. Assumption 3.1 means that the above function satisfied uniform Lipschitz condition. Most of common chaotic systems satisfies this condition, such as Lorenz system, Chen system, and Lü system. Although chaotic systems are bounded, the Lipschitz constant is difficult to determine. The case that L is unknown is considered in this paper.

In order to achieve self-synchronization of fractional-order system, we construct the controlled response system as follows:

$$ \begin{cases} D^{\alpha}\boldsymbol{y}(t)=A\boldsymbol{y}(t)+\phi(\boldsymbol{y}(t))+u(t,\boldsymbol{x}(t),\boldsymbol{y}(t)), & t\ne t_k,\ k=1,2,\cdots, \\ \Delta\boldsymbol{y}=\boldsymbol{y}(t_k^+)-\boldsymbol{y}(t_k^-)=B_k\boldsymbol{e}(t_k), & t=t_k, \end{cases} \tag{5} $$

where $\boldsymbol{y}\in\Re^n$ is the state vector of the system, $u(t,\boldsymbol{x}(t),\boldsymbol{y}(t))\in\Re^n$ is the adaptive controller. The discrete time set t_k satisfies $0<t_1<t_2<\cdots<t_k<\cdots$, $\lim_{k\to\infty}t_k=\infty$. And the initial time t_0 satisfies $0\le t_0<t_1$. $y(t_k^+)=\lim_{k\to t_k^+}y(t_k)$, $y(t_k^-)=\lim_{k\to t_k^-}y(t_k)$, and $y(t_k^-)=y(t_k)$ is assumed. $B_k=B_k^T$ are $n\times n$ gain matrices, $\boldsymbol{e}(t)=\boldsymbol{y}(t)-\boldsymbol{x}(t)=[y_1(t)-x_1(t),\ y_2(t)-x_2(t),\ \cdots,\ y_n(t)-x_n(t)]^T$ is synchronization error vector.

Substracting Eq. (3) from Eq. (5) yields error dynamical system as follows:

$$ \begin{cases} D^{\alpha}\boldsymbol{e}(t)=A\boldsymbol{e}(t)+\varphi(\boldsymbol{x}(t),\boldsymbol{y}(t))+u(t,\boldsymbol{x}(t),\boldsymbol{y}(t)), & t\ne t_k,\ k=1,2,\cdots, \\ \Delta\boldsymbol{e}=\boldsymbol{e}(t_k^+)-\boldsymbol{e}(t_k^-)=B_k\boldsymbol{e}(t_k), & t=t_k, \end{cases} \tag{6} $$

where $\varphi(\boldsymbol{x}(t),\boldsymbol{y}(t))=\phi(\boldsymbol{y}(t))-\phi(\boldsymbol{x}(t))$.

According to Assumption 3.1, we have

$$ \varphi(\boldsymbol{x}(t),\boldsymbol{y}(t))\le L\cdot\|\boldsymbol{e}(t)\|. \tag{7} $$

The synchronization problem is to design the adaptive controller and the parametric update law to achieve the asymptotical synchronization of the drive system (3) and the response system (5), that is, $\lim_{t\to+\infty}\boldsymbol{e}(t)=0$. However, to the author's knowledge, no mature theory is provided on the asymptotical stability of impulsive fractional-order system shaped like system (6).

To solve this problem, we will construct a new controlled response system as follows,

$$ \begin{cases} \dot{\boldsymbol{y}}(t)=A\boldsymbol{y}(t)+\phi(\boldsymbol{y}(t))+u(t,\boldsymbol{x}(t),\boldsymbol{y}(t))+N(\boldsymbol{x}(t)), & t\ne t_k,\ k=1,2,\cdots, \\ \Delta\boldsymbol{y}=\boldsymbol{y}(t_k^+)-\boldsymbol{y}(t_k^-)=B_k\boldsymbol{e}(t_k),\ t=t_k, \end{cases} \tag{8} $$

where $N(\boldsymbol{x}(t))=\dot{\boldsymbol{x}}(t)-D^{\alpha}\boldsymbol{x}(t)$, $\dot{\boldsymbol{x}}(t)$ and $D^{\alpha}\ \boldsymbol{x}(t)$ are from system (3). Then the following new synchronization error system can be obtained by systems (3) and (8)

$$ \begin{cases} \dot{\boldsymbol{e}}(t)=A\boldsymbol{e}(t)+\varphi(\boldsymbol{x}(t),\boldsymbol{y}(t))+u(t,\boldsymbol{x}(t),\boldsymbol{y}(t)), & t\ne t_k,\ k=1,2,\cdots, \\ \Delta\boldsymbol{e}=\boldsymbol{e}(t_k^+)-\boldsymbol{e}(t_k^-)=B_k\boldsymbol{e}(t_k),\ t=t_k, \end{cases} \tag{9} $$

Obviously, by constructing the response system (8), the synchronization of fractional-order chaotic system can be converted into the impulsive control of integral-order synchronization error system (9).

The adaptive impulsive controller is designed as follows:

$$ u(t,\boldsymbol{x}(t),\boldsymbol{y}(t))=-(\beta+\bar{L})\boldsymbol{e}(t), \tag{10} $$

where $\beta>0$ is a constant, the parameter $\bar{L}$ is used to approach the unknown parameter L, and its update law is given as

$$ \begin{cases} \dot{\bar{L}}=\gamma\|\boldsymbol{e}(t)\|^2, & t\ne t_k,\ k=1,2,\cdots, \\ \Delta\bar{L}=0, & t=t_k \end{cases} \tag{11} $$

where $\gamma>0$ is the adaptive rate.

To get the main conclusion, we give the following generalized Barbalat's lemma [24]:

Lemma 3.1. *Suppose a sequence t_k satisfies $0<t_1<t_2<\cdots<t_{k-1}<t_{k-1}<\cdots$ and $\lim_{k\to\infty}t_k=+\infty$, $\lambda=\inf_k\{t_k-t_{k-1}\}>0$. And suppose $f(t)$ is defined on the interval $[t_0,+\infty)$ and differentiable on the interval $[t_{k-1},t_k)$. If $f(t)$ and $\dot{f}(t)$ are uniformly bounded for k on the interval $[t_{k-1},t_k)$, that is, $\exists M_0$, $M_1>0$, $\forall t\in[t_{k-1},t_k)$, $k\in N$, one has $|f(t)|\le M_0$, $|\dot{f}(t)|\le M_1$, and the generalized integration $\int_0^{+\infty}f(t)dt$ is convergent, then $\lim_{t\to+\infty}f(t)=0$.*

By the theoretical proof, the main result is obtained as follows:

Theorem 3.1. *Let λ_A be the largest eigenvalue of $A+A^T$. If the adaptive impulsive controller (10) and the parametric update law (11) are adopted, as well as the following condition are satisfied:*

$$ (i)\lambda=\inf_k\{t_k-t_{k-1}\}>0; \tag{12} $$

$$ (ii)\lambda_{\max}((I+B_k)^T(I+B_k))\le 1; \tag{13} $$

$$ (iii)\frac{1}{2}\lambda_A-\beta<0,\ \text{that is},\ \beta>\frac{1}{2}\lambda_A; \tag{14} $$

then the synchronization error system (9) is asymptotically stable, that is, the impulsive controlled response system (5) and the drive system (3) asymptotically synchronize.

Proof. Let the Lyapunov candidate be

$$ V(\boldsymbol{e})=\frac{1}{2}\boldsymbol{e}^T\boldsymbol{e}+\frac{1}{2\gamma}(\bar{L}-L)^2. \tag{15} $$