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An effective method in calibrating nonlinear errors for phase-shifting interferometry

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ABSTRACT

In phase measurement or digital holography for phase-shifting interferometry, the key role is the variation of reference light wave and recover algorithm based on interferograms and reference phase, so the calculation result is directly affected by phase-shift accuracy. However, because of the errors of nonlinear and other random factors, it is difficult to control the actual phase-shifting amount accurately. In this paper, we aim to propose an efficient method for phase-shifting interferometry which does not require accurate initial estimation of phase-shift amounts, only a few pixels with several randomly shifted interferograms are sufficient for accurate extraction of phase information. This method has reduced the dependence of reference phase, and can obtain phase-shifting amount directly without using complex revised algorithm for correcting phase-shifting nonlinear errors.

1. Introduction

Phase-shifting interferometry (PSI) is a popular technique which has been widely used in optical testing [1,2], especially for the inspection of precision surfaces [3,4]. In this technique, the wave front reflected off or transmitted through the test object is measured by processing multiple interferograms obtained with different phase shifts. The conventional PSI requires a special constant phase shift (e.g., $\pi/2$ or $\pi/3$). However, errors of phase shifts are common in practice due to various reasons, and such errors can further cause substantial errors in the determinations of phase distributions. Therefore, plenty of efforts have been made to diminish the effects of phase shift errors including the nonlinear and irregular ones: Okada et al. [5] first proposed an iterative algorithm. Their research was a substantial extension of that previously proposed by Greivenkamp [6]. Lassahn et al. [7] presented two effective methods to calculate the phase steps and then evaluate the phase distribution. One of them was based on a polynomial approximation and a nonlinear least-squares fit. The other was based on a troublesome beam-shuttering method. The algorithm suggested by Farrell and Player [8,9] is based on Lissajou elliptic fitting. The method of Cai et al. [10] employs the spatial statistic of the interferograms.

In this paper, we aim to propose an efficient iterative least-squares algorithm for phase-shifting interferometry. The proposed algorithm makes the least squares converge accurately and efficiently, it does not require accurate initial estimation of phase-shift amounts, only several randomly shifted interferograms are sufficient for accurate extraction of phase information. By iterating the two steps: phase evaluation and phase shift correction, the phase distribution can be accurately reconstructed. The details of the proposed algorithm are described below.

2. Principle

The intensity of an interferogram can be expressed as

$$I_{ij} = A_j + B_j \cos(\varphi_j + \delta_i) \tag{1}$$

where I is the theoretical intensity of the interferogram, the subscript i denotes the ith phase-shifted image ($i = 1, 2, \ldots, N$), and j denotes the individual pixel locations in each image ($j = 1, 2, \ldots, M$). In this equation, A_j is the background or offset, B_j is the modulation amplitude, φ_j is the phase distribution of the wave-front to be measured, δ_i is the induced reference phase shift amount of each of N acquired frames, and M is the total number of pixels in each image frame. As mentioned in the previous section, δ_i is constant from frame to frame interferogram, and A_j, B_j and φ_j do not change at a point even if the phase shifts. These properties make it possible to calculate both the phase distribution φ_j and the amount of the phase shift δ_i by using an iterative method. The iteration has 2 steps in each cycle and different groups of unknowns can be released in relevant steps to make the problem solvable.

A. Initial calculation of the phase distribution

First of all, the unknown A_j is eliminated by defining the intensity difference J_{ij} such as:

$$I_{0j} = A_j + B_j \cos \varphi_j \quad (\text{assumed } d_0 = 0)$$

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$$J_{ij} = I_{ij} - I_{0j} = B_j \cos\varphi_j(\cos\delta_i - 1) - B_j \sin\varphi_j \sin\delta_i \tag{2}$$

Defining a new set of variables as:

$$b_j = B_j \cos\varphi_j \quad \text{and} \quad c_j = -B_j \sin\varphi_j$$

Eq. (2) is then worked out as:

$$J_{ij} = b_j(\cos\delta_i - 1) + c_j \sin\delta_i \tag{3}$$

The following step assumes that the phase shifts δ_i are known (provided by the arbitrary initial values or the previous iteration cycle). Then the least-squares error between theoretical and experimental interferograms, accumulated from all the images described by (3), can be rewritten as:

$$\varepsilon_j = \sum_{i=1}^{N}(J_{ij} - \tilde{J}_{ij})^2 = \sum_{i=1}^{N}(b_j(\cos\delta_i - 1) + c_j \sin\delta_i - \tilde{J}_{ij})^2 \tag{4}$$

In fact, ε_j represents the individual fitting error of the jth pixel summed over all N consecutive phase shifting. Now the necessary conditions to minimize ε_j are given by $\partial\varepsilon_j/\partial b_j = \partial\varepsilon_j/\partial c_j = 0$, which yields (5):

$$\begin{bmatrix} \sum_{i=1}^{N}(\cos\delta_i - 1)^2 & \sum_{i=1}^{N}(\cos\delta_i - 1)\sin\delta_i \\ \sum_{i=1}^{N}\sin\delta_i(\cos\delta_i - 1) & \sum_{i=1}^{N}\sin^2\delta_i \end{bmatrix} \begin{bmatrix} b_j \\ c_j \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N}\tilde{J}_{ij}(\cos\delta_i - 1) \\ \sum_{i=1}^{N}\tilde{J}_{ij}\sin\delta_i \end{bmatrix} \tag{5}$$

So the phase distribution φ_j may be obtained as:

$$\varphi_j = tg^{-1}\left(\frac{\sin\varphi_j}{\cos\varphi_j}\right) = tg^{-1}\left(-\frac{c_j}{b_j}\right) \tag{6}$$

B. Determination of the phase steps

With the phase distributions φ_j obtained from (6), the phase shift amounts δ_i can be determined in a similar but inverse way. Defining a new set of variables as:

$$b'_i = B_j(\cos\delta_i - 1) \quad \text{and} \quad c'_i = -B_j(\sin\delta_i)$$

Then the true values of δ_i will minimize the error ε_j that is defined as:

$$\varepsilon'_i = \sum_{j=1}^{M}(b'_i \cos\varphi'_j + c'_i \sin\varphi'_j - \tilde{J}_{ij})^2 \tag{7}$$

The condition should be imposed $\partial\varepsilon'_i/\partial b'_i = \partial\varepsilon'_i/\partial c'_i = 0$, produces the following matrix (8):

$$\begin{bmatrix} \sum_{j=1}^{M}\cos^2\varphi_j & \sum_{j=1}^{M}\cos\varphi_j \sin\varphi_j \\ \sum_{j=1}^{M}\sin\varphi_j \cos\varphi_j & \sum_{j=1}^{M}\sin^2\varphi_j \end{bmatrix} \begin{bmatrix} b'_i \\ c'_i \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{M}\tilde{J}_{ij}\cos\varphi_j \\ \sum_{j=1}^{M}\tilde{J}_{ij}\sin\varphi_j \end{bmatrix} \tag{8}$$

Therefore, the phase shift amounts δ_i is obtained as

$$\frac{c'_i}{b'_i} = \frac{-\sin\delta_i}{\cos\delta_i - 1} = ctg\frac{\delta'_i}{2} \Rightarrow \delta'_i = 2tg^{-1}\left(\frac{b'_i}{c'_i}\right) \tag{9}$$

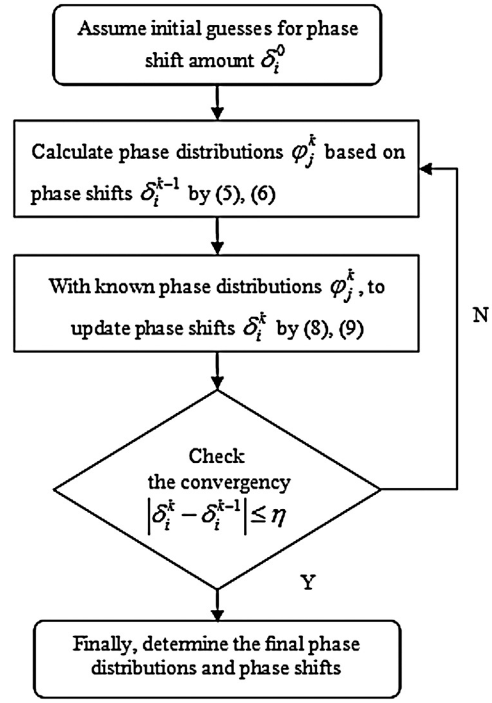

Fig. 1. Flow chart of numerical procedure with k stands for iteration number, while η stands for predetermined small constant.

C. Numerical procedures

The algorithm repeats steps A and B until the phase-shift values converge, necessary numerical procedures may be summarized as Fig. 1.

3. Configuration for simulation

This proposed phase-measuring algorithm is tested through computer simulation. In the simulation, the phase height of the object was defined as:

$$Z = 0.4 \times \sqrt{100^2 - (X - 150)^2 - (Y - 150)^2} \tag{10}$$

In order to compare with the real phase shifting amounts, we input interferogram fringe patterns with random phase shifting amounts which were generated by the computer. Fig. 2 shows the unwrapped phase distribution of the object.

The accuracy of the phase distribution calculation is directly governed by the accuracy of the phase shift amounts; therefore, the errors of the detected phase shift were evaluated in the simulation.

The errors were defined as:

$$e = \frac{1}{M}\sum_{i=1}^{M}|\delta_i - \delta_i^T| \tag{11}$$

where δ_i and δ_i^T are the algorithm-detected and true phase shift amounts respectively.

This simulation is repeated plenty times, and Table 1 shows only 8 cases of them, and the errors of phase shift of this algorithm were under 0.03 rad, the accuracy is very high. Furthermore, as the phase shift was created by computer randomly, it proved that the phase shift of proposed algorithm could be unequal and non-monotone.

In order to show the calibration of nonlinear errors in phase-shifting, we define the phase shift amount as $\delta_i = \delta_i^0 + \Delta\delta_i$, while