Optik 125 (2014) 2199–2204

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

A robust level set method based on local statistical information for noisy image segmentation

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ARTICLE INFO

Article history:
Received 10 May 2013
Accepted 11 October 2013

Keywords:
Active contour model
Image segmentation
Level set method
Statistical information
Noisy image

ABSTRACT

The paper presents an improved local region-based active contour model for image segmentation, which is robust to noise. A data fitting energy functional is defined in terms of curves and the energy terms which are based on the differences between the local average intensity and the global intensity means. Then the energy is incorporated into a level set variational formulation, from which a curve evolution equation is derived for energy minimization. And then the level set function is regularized by Gaussian filter to keep smooth and eliminate the re-initialization. By using the local statistical information, the proposed model can handle with noisy images. The proposed model is first presented as a two-phase level set formulation and then extended to a multi-phase one. Experimental results show desirable performances of the proposed model for both noisy synthetic and real images, especially with high level noise.

1. Introduction

Image segmentation has always been a difficult task in image processing and computer vision, the goal of which is to divide images into some meaningful subsets. Noise often occurs in all kinds of images: laser images, microscope images, sonar images, photoelectric images and all that, owing to the influence of the equipment of image transmission and processing, etc. [1,2]. The noise will bring about challenges in image segmentation.

In recent years, active contour models (ACM) implemented via level set methods have been successfully used in image segmentation [3–5]. The basic idea of ACM is to implicitly represent a contour as the zero level set of higher dimensional level set function, and formulate the evolution of the contour through the evolution of the level set function [6]. The pioneer work about ACM can be traced to Kass in 1988 [7], after which the approaches have developed in a variety of directions. Generally, the existing ACM can be broadly classified as either edged_based models [7–10] or region_based models [11–17].

Edged_based models [1–4] utilize image gradient information to stop the curve evolution. For this kind of models, it is not necessary to place a global constraint on the level set. The Geodesic active contour (GAC) model is one of the famous models in this class [8]. However, for some type of images, which object boundaries are weak or corrupted by noise, the edge-based models are likely to pass through the object boundary or produce spurious boundaries.

More recently, work has been focused on the region-based model. The region-based models [11–17] take advantage of a certain region descriptor (color, intensity, texture etc.). Therefore, they show better performance over the edge-based models in two aspects: First, the region-based models can obtain the promising results when handling with the weak boundaries. Second, the region-based models are less affected by the noise compared to the edge-based models, since they use the global region information.

One of the most famous region-based models is the CV model [11], which is a simplified Mumford-Shah model [12]. The CV model assumes that each region of the image has a statistically homogeneous intensity. And its energy functional is based on the difference of each pixel and the region intensity means. In fact, the images suffer from various types of artifacts such as intensity non-uniformity and noise. In such cases, the CV model fails to detect the object boundaries accurately.

However, the advent of the region_based active contour models [13–17] has had a significant impact on the intensity inhomogeneity. The local models assume that the intensities in a relatively small local region are separable. For instance, the RSF model, which is proposed by Li et al. [13,14], draws upon the local region intensity information in spatially varying local region, and hence does well in segmenting images with intensity inhomogeneity. Since the region_based models devote to the intensity inhomogeneity, they meet difficulties in segmenting images with high level noise.

In this paper, we focus on the need for the segmentation of the noisy images. Given that the CV model is based on the difference of the pixel and region average intensities, it would become invalid when it is polluted by noise. In this paper, we describe

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0030-4026/$ – see front matter © 2013 Elsevier GmbH. All rights reserved.
http://dx.doi.org/10.1016/j.ijleo.2013.10.026

an improved region_based CV model in a variational level set formulation to segment noisy images. The basic idea of the proposed model is to exploit the difference of local statistical information and global image information inside and outside of the contour to conduct an energy functional. When the curve is exactly on the boundaries of the object, the energy functional is minimized. The energy functional is incorporated into a level set functional to achieve its minimization. In addition, the proposed model is extended to multi-phase one. Furthermore, the Gaussian filtering processing [15,17] is adopted to regularize the level set functional and reduce the affect of the noise to a certain degree. Experimental results on some noisy synthetic and real images show the advantages of our model in term of efficiency and robustness. Moreover, compared with the well-known CV model, our model shows better performance on various noises and high level noise.

The remainder of this paper is organized as follows: A brief review of some well-known region_based models for image segmentation is given in Section 2. Then, Section 3 devotes to the proposed model in this paper. In Section 4, implement and experiment results are presented and analyzed. This paper is summarized in Section 5.

2. Backgrounds

Let Ω be the image domain, and I: $\Omega \rightarrow \Re$ be a gray level image. x of the $I(x)$ is the pixel in Ω. The goal of image segmentation is to divide the image into disjoint subregions $\Omega_1 \ldots \Omega_N$. On the basis of Mumford-Shah model, Chan and Vese proposed an active contour model (i.e. CV model [11]), energy of which in term of contour C can be written as:

$$\varepsilon^{CV}(c_1, c_2, C) = \lambda_1 \cdot \int_{outside(C)} (I(x) - c_1)^2 \, dx + \lambda_2 \cdot \int_{inside(C)} (I(x) - c_2)^2 \, dx + \nu \left| C \right| \tag{1}$$

where *outside* (C) and *inside* (C) represent the regions outside and inside the curve C, respectively. c_1 and c_2 designate two constants that approximate the image intensities in *outside* (C) and *inside* (C), respectively. λ_1, λ_2, are fixed constants. The third term is the length of curve C for regularization. For parameter $\nu \geq 0$, if we have to not detect smaller objects (like points, due to noise), ν has to be larger [11].

In order to deal with intensity inhomogeneity, Li et al. brought forward a region_based active contour model that draws upon intensity information in local regions at a controllable scale (RSF) [13,14]. Considering the two-phase model, the energy functional of the RSF model is expressed as:

$$\varepsilon^{RSF}(\phi, f_1, f_2) = \lambda_1 \int \left[\int K_\sigma(x-y) \cdot \left| I(y) - f_1(x) \right|^2 \cdot H(\phi(y)) \, dy \right] dx + \lambda_2 \int \left[\int K_\sigma(x-y) \cdot \left| I(y) - f_2(x) \right|^2 \cdot H(-\phi(y)) \, dy \right] dx + \nu \int \left| \nabla H(\phi(x)) \, dx \right| + \mu \int \frac{1}{2} (\nabla \phi(x) - 1)^2 \, dx \tag{2}$$

where λ_1, λ_2, are positive constants which govern the tradeoff between the first term and the second term. $f_1(x)$, $f_2(x)$ are two values that locally approximate image intensities on the two sides of C. The size of the local region can be controlled by the scale parameterσ of kernel function K_σ. It is chosen as a Gaussian kernel $K_{\sigma(x-y)}$, which decreases drastically to zero as y goes away from x:

$$K_\sigma(u) = \frac{1}{2\pi\sigma^2} e^{-|u|^2/2\sigma^2} \tag{3}$$

The third term is the length term that regularizes the zero level contours, and the last term is the penalty term that penalizes the deviation of the level set function from a signed distance function.

However, inspired by the thought of the RSF model, we propose an improved active contour model based on local intensity information for noisy image segmentation, which is an improved CV model.

3. The proposed model

Let Ω be the image domain, which can be partitioned to a set of disjoint regions noted as $\left\{ \Omega_i \right\}_{i=1}^{N}$. N is the number of the regions, and $\Omega = \cup_{i=1}^{N} \Omega_i$, for $i \neq j$, $(\Omega_i \cap \Omega_j = \phi)$.

For the CV model, the energy is based on the difference between each pixel and the average intensity of the region. However, when the image is polluted by noise, this model may result in accurate segmentation results. In our model, we utilize the local intensity average instead of the single pixel to establish the energy functional. In that case, the proposed model is less affected by the noise.

3.1. Level set formulation: two-phase model

Let C be a closed contour in the image domain, which separates the domain into two parts: Ω_1 *outside* (C) and Ω_1 *inside* (C). *inside* (C) and *outside* (C) represent the regions inside and outside the contour C, respectively. Considering the two-phase model, we define the following two-phase model in term of contour C:

$$\varepsilon(c_i, f_i, C) = \varepsilon_1 + \varepsilon_2 = \int_{\Omega_1} (f_1(x) - c_1(x))^2 \, dx + \int_{\Omega_2} (f_2(x) - c_2(x))^2 \, dx \tag{4}$$

where $c_1(x)$ and $c_2(x)$ are constants which stand for the average intensities of Ω_1 and Ω_2, respectively. $f_1(x)$ and $f_2(x)$ indicate the weighted intensity means of the neighborhood region partitioned by the contours. When the energy is minimized, the curve is exactly on the object boundaries.

In the level set formulation, active contours are represented by the zero level set $C(t) = \left\{ (x, y) \middle| \phi(x, y, t) = 0 \right\}$ of a level set function ϕ. Then Ω_1 and Ω_2 can be represented as the two regions outside and inside the zero level set of ϕ, i.e., $\Omega_1 = \{\phi > 0\}$ and $\Omega_2 = \{\phi < 0\}$. Using Heaviside function H, the energy in Eq. (4) can be expressed as an energy functional in terms of ϕ, c_i and f_i as below:

$$\varepsilon(c_1, c_2, f_1, f_2, \phi) = \sum_{i=1}^{2} \int (f_i(x) - c_i(x))^2 M_i(\phi(x)) \, dx \tag{5}$$

where $M_1(\phi(x)) = H(\phi(x))$, $M_2(\phi(x)) = 1 - H(\phi(x))$, in practice, Heaviside function $H(\phi(x))$ and its derivative Dirac function $\delta(\phi(x))$ are approximated respectively as smooth functions $H_\varepsilon(\phi(x))$ and $\delta_\varepsilon(\phi(x))$, defined by:

$$\begin{cases} H_\varepsilon(x) = \dfrac{1}{2} \left[1 + \dfrac{2}{\pi} \arctan \left(\dfrac{x}{\varepsilon} \right) \right] \\ \delta_\varepsilon(x) = H'(x) = \dfrac{1}{\pi} \cdot \dfrac{\varepsilon}{\varepsilon^2 + x^2} \end{cases} \tag{6}$$

where the parameter ε can be set to 1.0 as in [11]. And to keep the notions simple, we still write $H(\phi)$ and $\delta(\phi)$ instead of $H_\varepsilon(\phi)$ and $\delta_\varepsilon(\phi)$.

c_i, f_i are the region average intensities and local intensity means, respectively, which are defined by the following functions:

$$c_i(x) = \frac{\int I(x) M_i(\phi(x)) \, dx}{\int M_i(\phi(x)) \, dx} \tag{7}$$