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Property of robustness to size and its realization on fractal dimension

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ABSTRACT

Images in one class often have varied sizes due to different imaging system. Thus it will provide convenience to image classification if the indicator used in the classification is robust to the size of images. We regard the robustness to size of image as a property of image indicator. The property means that images from one class have small variance with the sizes, and is different from such traditional properties as the robustness to scale, rotation and illumination. Fractal dimension is an indicator which has the three traditional properties. We realize the property on fractal dimension in the statistical sense by modifying differential-box counting method. Tests on two classes of images demonstrate the effectiveness of the modifications. Tests on scaling process give a standard of FD' robustness as 0.0611, and experiments on both the two class and four sets of images show the statistical validity of the standard and verify the realization. An indicator with this property can be a tool for the classification.

1. Introduction

Images in one class are often given by many imaging systems. In general cases, such systems output images of varied sizes. This confronts both image classification and other image analysis tasks with the challenge how to abstract the class information. Indicators of images can be very helpful, if they can be robust to size of images. If a kind of indicator has a small variance with sizes of images in one class, it can be a tool to determine whether an image is in the class. In other words, the image is very possible to be in the class, if its indicator is similar to those indicators of images in the class. Two indicators are similar if their difference is below the variance.

Indicators have been developed for many tasks, such as fractal dimension (FD) [1–3] for image segmentation [4–7]. A good indicator is expected to be robust to three transformations which the imaging systems often endure, including scale, rotation, and illumination. FD is an indicator robust (invariant) to the three transformations. In practice the estimate of FD (EOFD) is not invariant but only robust to the transformations because of the finiteness of sampling rate; in this sense we use the word "robust" instead of "invariant". However, the robustness to size is unnoticed, and EOFD is not robust to size of image. In fact, size differs itself from the three transformations. For an example, let's image a graph of sine function, and then scale affects the image by sampling rate, while size affects the image by how long the graph is imaged at a sampling rate. In other words, scale affects the imaging process by changing sampling rate of an imaged scene, while size affects the process if the scene is still in the same class and the sampling rate is not changed. We regard this robustness to size as a property of image indicator.

The classification is similar to the feature retrieving task in [8], where the authors show successfully the usefulness for EOFD to the task. Note that images of small size such as 4×4 make sense in the tasks, and then EOFD's behavior on images of small size is not trivial. For convenience, hereafter we do not distinguish FD from EOFD if not pointing out. FD is also useful in image segmentation [4–7], image coding [9], edge detection [10,11], biometrics technique [12], network [13], corn progress detection [14] and maritime target detection [15].

2. Related concepts of FD

To estimate EOFD, in [16] ε blanket method is proposed. In [17,18] fractional Brownian random filed method is developed. In [19] the differential box-counting method (DBC) is developed based on the work in [20]. We prefer DBC only because the method is similar to the definition of Hausdorff dimension.

In [21] the size problem is noted in the sense of data sample size. The work is toward another goal, expanding FD's effectiveness on more data sets. In [22,23] good revisions are presented to promote successfully the accuracy of FD. In [24,25] the lacunarity, a measure of texture of a fractal object, is discussed, to be an indicator to reflect the complex geometry. In [26] a good parallel implementation of DBC is reported.

Through the definition of FD, if one covers the measured object completely with the number $N(\varepsilon)$ of balls of radius at most ε, and

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if $N(\varepsilon)$ always grows with ε^{-D} as ε approaches zero, then the measured object has FD as D:

$$N(\varepsilon) \propto \varepsilon^{-D} \quad (1)$$

Consider the image of size $M \times M$ as a three dimensional space with (x, y) denoting two dimensional position and with gray level denoted as the third dimension z. The (x, y) plane is partitioned into grids of size $\varepsilon \times \varepsilon$. Each grid is composed by boxes of size $\varepsilon \times \varepsilon \times \varepsilon$. In the (i, j) grid let the minimum and maximum gray level of the image fall into the k^{th} box and the l^{th} box, respectively. Then the number of boxes needed to cover the image surface over the (i, j) grid is given by:

$$N_{i,j}(\varepsilon) = l - k + 1. \quad (2)$$

Here the constant 1 is intended to cover the box where $l = k$. Or calculate first the difference between the minimum and maximum gray level in the grid, and then correspond the difference to the $(l - k)$ in Eq. (2) [16] by scaling the difference by ε. In this paper we choose this way. And one obtains $N(\varepsilon)$ in Eq. (1) by:

$$N(\varepsilon) = \sum_{i,j} N_{i,j}(\varepsilon). \quad (3)$$

$N(\varepsilon)$ is clues of ε. Then we obtain FD of the image by the least-squares linear fit of $\lg(N(\varepsilon))$ against $\lg(1/\varepsilon)$.

3. Methods

We think that two problems in DBC are responsible for FD's vulnerability to size as follows.

The way of cover: Consider the case where the size of an image contains prime number, e.g., M is prime number. Given that every box is of size $s \times s \times s$ and s takes 2 as its minimum, boxes cannot cover the entire (x, y) plane except the case where s is increased to the prime number, i.e., part of information in the image has to be lost. Images from the same class should have common information which labels the class. The lost caused by incomplete cover may reduce the information's effectiveness on the classification. In [27] the authors note this problem of incomplete cover, but they take the values in the uncovered region as zero.

The constant 1: The constant in Eq. (2) has a definite physical meaning that an area needs at least 1 box to cover [19]. We write the constant as c, to make Eq. (2) as follows: $N_{i,j}(\varepsilon) = l - k + c$. But in the terms of programming, by Eq. (2) we know that the constant has two effects. First, the constant strengthens the numerical stability when $\lg(N(\varepsilon))$ is calculated, since that Eq. (3) will give $N(\varepsilon) = 0$ if $l = k$ in each grid in the case of $c = 0$. The value of c should not be too small for the stability of $\lg(N(\varepsilon))$. Second, it affects the value domain of FD if we consider FD as a function of the constant. With ε decreases, $(l - k)$ increases with ε^{-D} but c does not increase, to make $N_{i,j}(\varepsilon)$ and even $N(\varepsilon)$ not satisfy Eq. (1). The constant brings about deviation from FDs in the case of $l \neq k$; and the smaller c is, the smaller the deviation is.

To enable FD with the robustness, we cover the surface of the image as follows. Consider an image of size $M \times N$. First use boxes of size $\varepsilon \times \varepsilon \times \varepsilon$ to cover the surface at most extent, in the way the same as that in DBC. Then for the uncovered area if existing, use a box to cover the entire surface. This way of cover adds a contribution to $N(\varepsilon)$ in DBC at a given ε.

The modification is effective, as demonstrated in both Figs. 1 and 2. In Fig. 1, for the class of single gray level image where the sizes increase from 2^3 to 2^{12}, DBC gives FDs' variance as about 0.07 while the modified DBC gives FDs' variance as about 0.03. In Fig. 2, for the class of grating figure where the sizes increase from 2^3 to 2^{12}, DBC gives the variance as about 0.08 while the modified DBC gives the variance as about 0.03. Thus the tests demonstrate the effectiveness of the modification. Note that images of small size make sense as shown in [8].

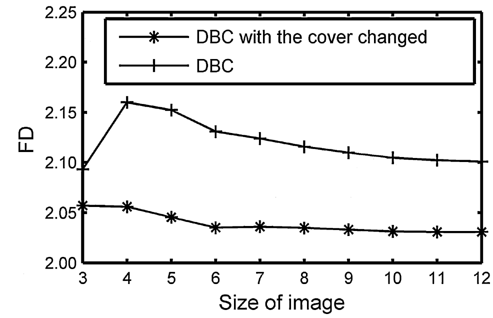

Fig. 1. Effect of the modified way of cover on single gray level images.

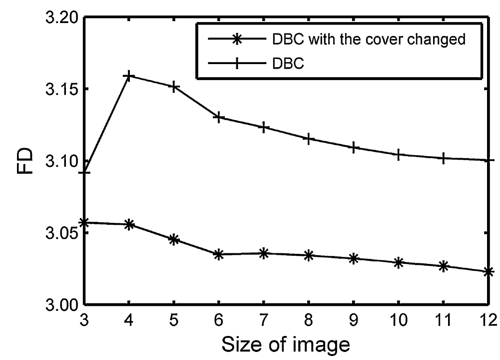

Fig. 2. Effect of the modified way of cover on grating figures.

By changing the value of the constant c at a given ε, we obtain the effect which the constant takes on FD. First, we *obtain* $N(\varepsilon)$ by letting $c = c_0$. Second, we obtain $N_c(\varepsilon)$ by letting $c = c_1$. Denote $f(c) = N(\varepsilon) - N_c(\varepsilon)$, i.e., the changing decreases $N(\varepsilon)$ by function $f(c)$. By Eq. (2) we know that $f(c)$ is proportional to c; moreover, $f(c)$ is a constant for $N(\varepsilon)$ at a given ε, because that the way of cover is the same. Assume $c_1 > 0$ and $c_0 > 0$, then the function is positive if $c_0 > c_1$, negative if $c_0 < c_1$. Then if c is increased from c_0 to c_1, FD obtained from c_0 would be bigger than that from c_1. The function's effect is small if every $(l - k)$ in Eq. (2) is big enough, because in this case c contributes smaller to $N(\varepsilon)$ than $(l - k)$. Whereas $(l - k)$ is not always so big, the effect raises the problem which value of c is appropriate for FD.

According to our experience, no image has $N(\varepsilon)$ less than the $N(\varepsilon)$ of a single gray level image, where Eq. (2) gives only c as contribution for every ε. In theory FD of single gray level image is 2. The value of c should keep this property. In practice EOFD is not exactly 2; a deviation varying with the way of cover and varying with the value of the constant exists. Hence this property is supported if FD of single gray level image is close to 2, where the deviation varies with both the way of cover and the constant. DBC gives the deviation as demonstrated in Fig. 1, where the size is from 2^3 to 2^{12}, and where DBC gives FDs from 2.09 to 2.16 while the modified DBC gives FDs from 2.03 to 2.06.

Consider the case where $N(\varepsilon)$ of an image is big enough. It is expected that FD of the image can be close to 3 since that $N(\varepsilon)$ for every ε achieves its maximum. According to our experience, grating figure is such an image if it takes only two values, 0 and 255, as its pixel value. The value of c should also keep the property that FDs of grating figures are almost 3.00. This property is also supported by DBC, with a deviation demonstrated in Fig. 2. Though it is seen from both Figs. 1 and 2 that the modification to the way of cover decreases the deviation, this effect on accuracy is not concerned in this paper because robustness instead of accuracy is vital for image