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Dispersive optical solitons by Kudryashov's method

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ABSTRACT

This paper studies the dynamics of dispersive optical solitons that is modeled by the fourth order nonlinear Schrödinger's equation and Schrödinger–Hirota equation, the latter of which is considered with power law nonlinearity. Kudryashov's method is applied to obtain soliton solutions to the model equations. These results and the solution methodology makes a profound impact in the study of optical solitons.

1. Introduction

Optical solitons is a major area of research in the area of nonlinear optics. These solitons form the fabric of our daily lives in the Internet word and other forms of global electronic communications across trans-continental and trans-oceanic distances. The study of solitons in the context of optics is going on the past few decades and overwhelming results have been reported in several journals. One of the main aspects in the study of optical solitons is the issue of integrability. There are two types of models that will be considered in this paper. They are the nonlinear Schrödinger's equation (NLSE), in Kerr law medium, with fourth order dispersion (4OD) and the Schrödinger-Hirota equation (SHE) that will be considered with power law nonlinearity.

SHE is obtained from NLSE with Kerr law nonlinearity and third order dispersion (3OD) by the aid of Lie symmetry analysis, with a valid approximation. The details are obtained earlier [25,29,30]. Both NLSE and SHE are nonlinear partial differential equations of *S*-type. SHE was studied earlier on several occasions [25,28–30]. The exact 1-soliton solutions were obtained. In fact, bright, dark and singular soliton solutions were addressed in this context. This paper will again focus on the integrability aspect of SHE and NLSE with 4OD. However, integration architecture will be different. In fact, Kudryashov's method, that is also known as modified truncated expansion method, will be applied to extract soliton solutions SHE and NLSE with Kerr law nonlinearity and 4OD.

The aim of this paper is to extract the new exact special solutions of the model equations after implementing Kudryashov method. The computer symbolic systems such as *Maple* and *Mathematica* allow us to perform merciless and unforgiving calculations. The paper is arranged as follows. In Section 2, we describe briefly the Kudryashov method. In Sections 3 and 4, Kudryashov's method is applied to extract the soliton solutions.

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2. Kudryashov's method

Let us present the algorithm of modification of truncated expansion method (Kudryashov method) for finding exact solutions of nonlinear PDEs. We consider the nonlinear PDE in the following form:

$$E_1(u, u_t, u_x, u_{xx}, \ldots) = 0. \tag{1}$$

Using traveling wave $u(x, t) = U(\xi)$, $\xi = kx - \omega t$ carries Eq. (1) into the following ordinary differential equation (ODE):

$$E_2(U, -\omega U_\xi, kU_\xi, k^2 U_{\xi\xi}, \ldots) = 0. \tag{2}$$

The Kudryashov method contains the following steps.

Step-I. We look for exact solution of Eq. (2) in the form

$$U = \sum_{i=0}^{N} a_i Q^i(\xi) \tag{3}$$

where $a_i (i = 0, 1, \ldots, N)$ are constants to be determined later, such that $a_N \neq 0$, while $Q(\xi)$ has the form

$$Q(\xi) = \frac{1}{1 + \rho \exp(\xi)} \tag{4}$$

a solution to the Riccati equation

$$Q_\xi = Q^2 - Q$$

where ρ is arbitrary constant.

Remark: This Riccati equation also admits the following exact solutions [31]:

$$Q_1(\xi) = \frac{1}{2}\left(1 - \tanh\left[\frac{\xi}{2} - \frac{\varepsilon \ln \xi_0}{2}\right]\right), \quad \xi_0 > 0$$

and

$$Q_2(\xi) = \frac{1}{2}\left(1 - \coth\left[\frac{\xi}{2} - \frac{\varepsilon \ln \xi_0}{2}\right]\right), \quad \xi_0 < 0,$$

more general solutions are presented in reference [31].

Step-II. We determine the positive integer N in Eq. (3) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (2).

Step-III. We substitute Eq. (3) into Eq. (2), and calculate all the necessary derivatives $U_\xi, U_{\xi\xi}, \ldots$ of the unknown function $U(\xi)$ as follows:

$$U_\xi = \sum_{i=1}^{N} a_i i (Q - 1) Q^i, \tag{5}$$

$$U_{\xi\xi} = \sum_{i=1}^{N} a_i i [(1 + i)Q^2 - (2i + 1)Q + i] Q^i, \tag{6}$$

and so on. Substituting Eqs. (3), (5) and (6) into Eq. (2), we obtain the polynomial

$$E_2[Q(\xi)] = 0. \tag{7}$$

Step-IV. Collecting all the terms of the same powers of the function $Q(\xi)$ in the polynomial (7) and equating them to zero, we obtain a system of algebraic equations which can be solved by computer programs such as Maple and Mathematica to get the unknown parameters a_i, k and ω. Consequently, we obtain the exact solutions of Eq. (1).

3. NLSE with 4OD

In this section, we study the NLSE with Kerr nonlinearity and 4OD [27] in the following form:

$$iq_t + aq_{xx} - bq_{xxxx} + c|q|^2 q = 0. \tag{8}$$

The function q is a complex valued function of the spatial coordinate x and the time t. The function q is a sufficiently differentiable function. With $b = 0$, Eq. (8) reduces to NLSE with Kerr law nonlinearity.

Since $q = q(x, t)$ in Eq. (8) is a complex function we suppose that

$$q(x, t) = U(\xi) e^{i(\alpha x + \beta t)}, \quad \xi = i(kx - \omega t), \tag{9}$$

where α, β, k and ω are constants, all of them are to be determined.

On substituting these into Eq. (8) yields

$$q_t = i\left(-\omega U' + \beta U\right) e^{i(\alpha x + \beta t)}, \tag{10}$$

$$q_{xx} = -(k^2 U'' + 2k\alpha U' + \alpha^2 U) e^{i(\alpha x + \beta t)}, \tag{11}$$