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Nonlinear dynamics of an optical phase locked loop in presence of additional loop time delay

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ABSTRACT

The dynamics of an optical phase locked loop (OPLL) with first order loop filter having inherent loop time delay is investigated. In the presence of delay, the system is modeled as a third order autonomous system. In the out of lock condition or during the process of locking, the dynamics of the system is highly nonlinear and different nonlinear phenomena, like limit cycle oscillation, period doubling, chaotic oscillations etc., may be observed with the variation of design parameters. Applying the techniques of the nonlinear dynamics, we have calculated the effects of the inherent loop time delay in determining the state of the loop. The analytical results predicting the parameter values for stable and unstable region of operation are obtained using the quasi-linear Routh–Hurwitz method. The parameter range required for the onset of chaotic oscillations is estimated by Melnikov's global perturbation method. The predicted results are in agreement with those obtained by numerical integration of system equations.

1. Introduction

PLLs are important parts of modern electronic communication and control systems. They are used in demodulation of frequency and phase modulated signals, frequency synthesis, data and clock recovery circuits etc. Recently, increased interests are being shown on the nonlinear dynamics of PLLs, since they could be used as generators and synchronizers of dynamical chaos [1,2]. The idea of developing PLL based high frequency oscillators with chaotic frequency and phase modulation is rather attractive for realizing the data transfer technique using chaos shift keying. The basic building blocks of a PLL are (as shown in Fig. 1) a phase detector (PD), a low pass filter (LPF), and a voltage controlled oscillator (VCO). A PLL is said to be in a phase locked state when the loop phase error attains a constant value in the face of an un-modulated input signal. However, PLLs having inherent additional phase shift can jump into a false locked state for certain loop parameters and one gets chaotic oscillations in the PLL dynamics [3–6].

Over the last few decades, the frequency range of application of PLLs has extended from lower RF to microwave frequencies and beyond that. With the advancement of optoelectronic technology, PLL systems have been designed in the optical frequency range also [7]. As such OPLLs have huge application potential and several works on OPLLs have been reported in the literature [8–10]. In future, OPLLs would be used in different applications, such as optical phase modulation, dispersion compensation in electrical domain, fast reconfigurable optical networks, optical sensor etc. [9,10]. OPLL can also be used to generate highly stable microwave carriers, which can be used in optical communication [8]. However, as its electronic counterpart, an optical system is associated with some inherent time delay depending on the physics of time response of different blocks used in the circuit.

A functional block diagram of a conventional PLL with inherent delay is shown in Fig. 2. In the corresponding block diagram of an OPLL (shown in Fig. 3), the PD and the local reference oscillator are replaced by a photo detector based circuit and a current controlled semiconductor laser oscillator, called laser local oscillator (LO) respectively. Here PD circuit consists of an optical coupler, which combines the two optical signals, two photodiodes and an amplifier. Photodiodes generate electrical signals corresponding to the difference frequency of two optical signals and it is amplified by the amplifier. The signal obtained from the amplifier is applied to the current controlled laser source through a loop filter (LF). The inherent time delay in the loop is modeled by a constant time delay block as shown in the figure. Depending on the nonlinearity and delay present in the circuit, the dynamics of the OPLL may sometimes become unstable and chaotic. Under this condition

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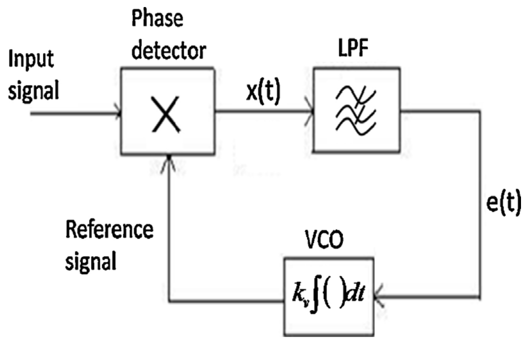

Fig. 1. Basic block diagram of PLL.

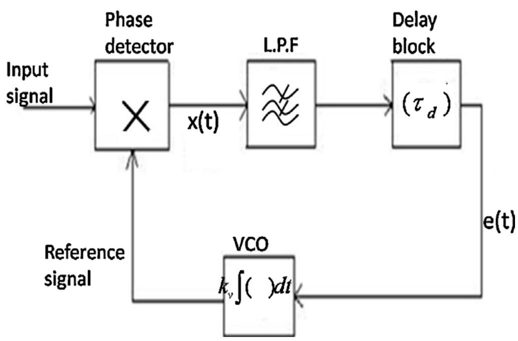

Fig. 2. Block diagram of PLL in presence of delay.

chaotically modulated optical signal would be obtained from the OPLL. Recently in some works [11–13], different experimental and theoretical studies on optical communication based on chaotic laser signal have been reported. It has been shown that by using chaotic optical signals the enhancement of communication bandwidth with increased security can be obtained. In this respect a chaotically oscillating OPLL could be a useful source of chaotic optical signals as obtained from the reference oscillator of the OPLL. This study would also be used to evaluate the amount of time delay that could be tolerated by the loop for stable operation and thus help the designer to implement a chaos-free OPLL. On the other hand, incorporating additional time delay deliberately it would be possible to get chaotically modulated optical signals from an OPLL.

In this paper, we have studied the dynamics of the OPLL by examining the system equation quasi analytically as well as through numerical integration. By applying the Routh–Hurwitz stability criterion to the linear approximation based characteristic equation of the closed loop system, the stable and unstable zones of loop phase error are obtained. The unstable zone of loop operation has been critically examined to find the parameter space for limit cycle oscillations of single and multiple periods. By using Floquet theory [14,15], the parameter zone in which period-1 oscillation bifurcates to period-2 oscillation has been obtained. To obtain the parameter range for chaotic oscillation of phase error in the unstable region, we have applied Melnikov's method [16–19] of global perturbation and a range has been obtained.

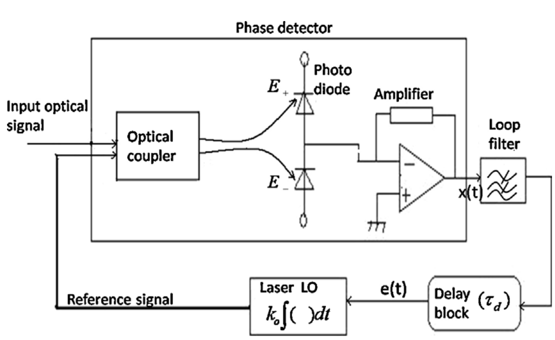

Fig. 3. Block diagram of OPLL in presence of delay.

The whole study is divided into the following sections. In Section 2, the OPLL system has been described and by considering the proper transfer functions of the LF and the inherent loop time delay, the system equation has been derived. The system equation has been studied analytically thoroughly in Section 3. This section consists of three subsections. Firstly, by using Routh–Hurwitz stability analysis, the condition of stable phase error and the condition of oscillation of phase error have been determined. Secondly, by using the theory of limit cycle bifurcation, the zone, in which period-1 oscillation bifurcates to period-2, 4 etc. has been determined. Then by using Melnikov's technique, the condition of chaotic oscillation of the phase error has been obtained. In Section 4, numerical simulation results depending on different values of parameters and time delay have been obtained by solving the system equations by Runge–Kutta technique numerically. Finally in Section 5, some concluding remarks have been given based on analytical and simulation results.

2. Derivation of system equation for OPLL in presence of inherent time delay

Basic configuration of an OPLL considering inherent time delay is shown in Fig. 3. We consider the amount of inherent time delay as τ_d and its effect is taken by a network of transfer function, $F_d(s) = e^{-s\tau_d}$, in the frequency domain, where, s is the complex frequency. In practical situations the time delay is small compared to the loop filter time constant and so the transfer function representing time delay can be approximated as, $F_d(s) = \left(1 - s\tau_d/2\right) / \left(1 + s\tau_d/2\right)$.

2.1. Response of photo diode based PD

Let the input optical signal and reference optical signal obtained from laser LO be denoted as,

$$E_1 = \sqrt{P_1}\sin\theta_i(t) \tag{1a}$$

$$E_2 = \sqrt{P_0}\cos\theta_0(t) \tag{1b}$$

where P_i, P_0 and θ_i, θ_0 are the powers and phases of two optical signals. Now the optical coupler produces two signals corresponding to sum and difference of the signals applied to it, given by,

$$E_+ = \frac{1}{\sqrt{2}}(E_1 + E_2) \tag{2a}$$

$$E_- = \frac{1}{\sqrt{2}}(E_1 - E_2) \tag{2b}$$

These two signals are made to incident on two photodiodes and they produce electrical current according to the relation, $i_d = R_p\left|E^2\right|$, where R_p is the responsivity (in Amperes per Watts) of photodiode. The total current (i), applied to the amplifier is the difference of two diode currents and can be easily calculated as,

$$i = i_{d1} - i_{d2} = 2R_p\sqrt{P_iP_0}\sin\ \varphi \tag{3}$$

here $\varphi = \theta_i(t) - \theta_0(t)$, is the phase error between the phases of the input optical signal and the signal from the laser LO. If k_a be the gain of the amplifier used in PD, then the current obtained from the PD is

$$i_p = k_a i = k_p\sin\ \varphi \tag{4}$$

where $k_p = 2k_aR_p\sqrt{P_iP_0}$, is the sensitivity of the PD.