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A four-wing and double-wing 3D chaotic system based on sign function

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ABSTRACT

Many four-wing chaotic systems have been developed based on cross product or quadratic operations. Differently, we construct a three-dimensional chaotic system generating four-wing or double-wing attractors by virtue of sign function. Dynamical properties such as equilibrium points, Poincaré map, Lyapunov exponent spectra, Hopf bifurcations and bifurcation diagrams of the system are theoretically and numerically analyzed. Results of mathematical analyses and simulation tests indicate that the proposed chaotic system can keep chaotic to generate four-wing or double-wing attractors within a large scope of parameters. The system also shows hyperchaotic behaviors in some parameter range. Besides, circuit implementation of the chaotic system is studied. That proves the system is physically realizable.

1. Introduction

Chaos, characterized by its deterministic, unpredictable features and extremely sensitive dependence on initial conditions, stems from nonlinear systems [1]. It can be applied in the fields of data communications, signal detection, cryptology and so on. Since the Lorenz chaotic system [2] was proposed in 1963, many chaotic systems related to the Lorenz system had been reported [3–6]. For engineering applications, more complex chaotic attractors will guarantee more randomness and more security. Therefore, many researchers pay attentions to chaotic systems generating complicated attractors, such as multi-layer attractors [7], multi-scroll attractors [8–11], and multi-wing attractors [12–23].

Dong et al proposed a three-dimensional system which has complex multilayer chaotic attractors [7]. Besides, many multi-scroll chaotic systems have been developed. Yu and Qiu studied the dynamical properties of multiple-scroll chaotic attractors [8]. Yu and Lü implemented an *n*-scroll chaotic system based on a general Jerk circuit [9]. Sara et al. designed a system that can generate four-scroll hyperchaotic and four-scroll chaotic attractors [10]. Jesus et al. proposed a system generating multiple scrolls in multiple directions (two- and three-directions) on phase space [11]. The generation of multi-wing chaotic systems is also one of hot topics. Lü et al. established a real four-wing chaotic system [12]. Many chaotic systems generating multi-wing attractors were subsequently proposed [13–20], including some multi-wing four-dimensional hyperchaotic and chaotic systems [21–23]. Although many multi-wing chaotic systems have been developed, most of the systems are realized by linear functions, cross terms and square operations, and other functions are rarely adopted. Based on sign function, we designed a simple multi-wing three-dimensional chaotic system. It can generate two-wing or four-wing attractors in a large range of parameters. That makes it a possible candidate for many engineering fields, such as secure communications, information processing, encryption, etc.

This paper is organized as follows. In Section 2, mathematic model and dynamical behaviors such as equilibrium points, Poincaré map, Lyapunov exponent spectra and bifurcation diagrams of the three-dimensional chaotic system are discussed in details. In Section 3, different chaotic attractors of the system orbits are listed with varied system parameters and initial conditions. The circuit diagram and Multisim 10 simulation results of the system are shown in Section 4. Finally, Section 5 draws the conclusions.

2. A new multi-wing three-dimensional chaotic system

A new four-wing three-dimensional chaotic system is established by the following equations.

$$\begin{cases} \dot{x} = -ax + yz + d\mathrm{sgn}(y) \\ \dot{y} = by - xz \\ \dot{z} = -cz + xy \end{cases} \tag{1}$$

where $a > 0$, $b > 0$, $c > 0$, and d is non-zero. When $a = 3$, $b = 5$, $c = 20$, $d = 5$, the corresponding chaotic attractors of the system (1) are

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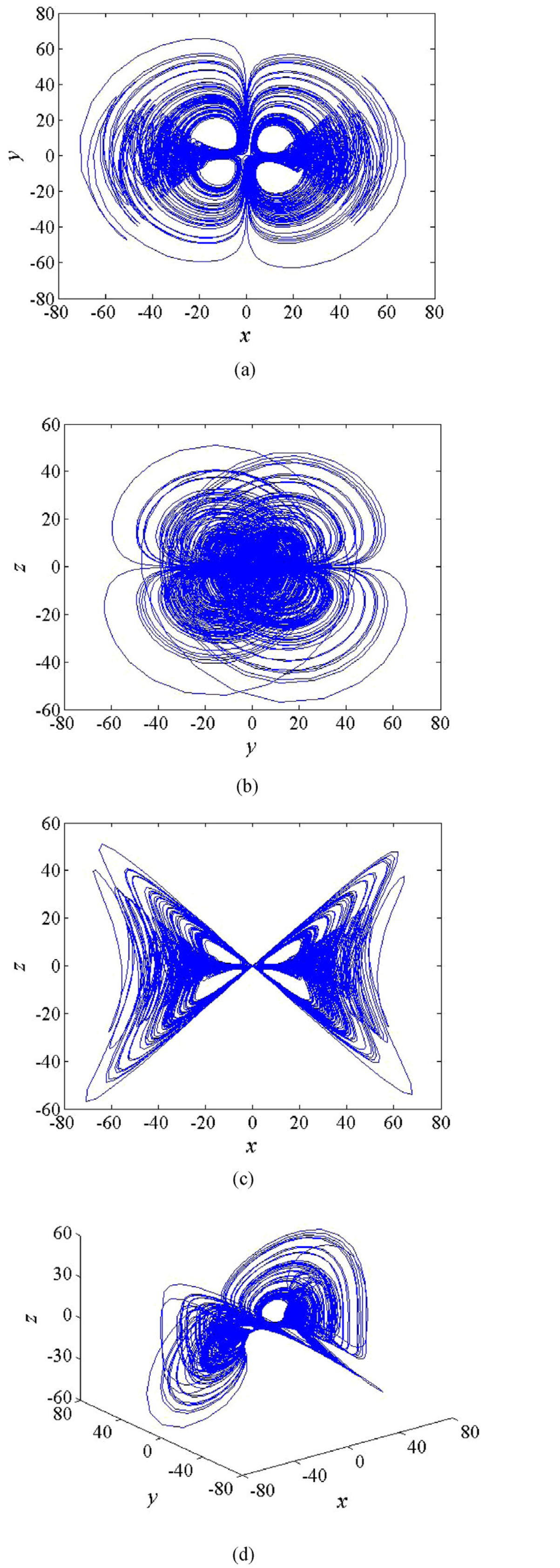

Fig. 1. Phase diagrams when $(a, b, c, d) = (3, 5, 20, 5)$: (a) $x-y$; (b) $y-z$; (c) $x-z$; (d) $x-y-z$.

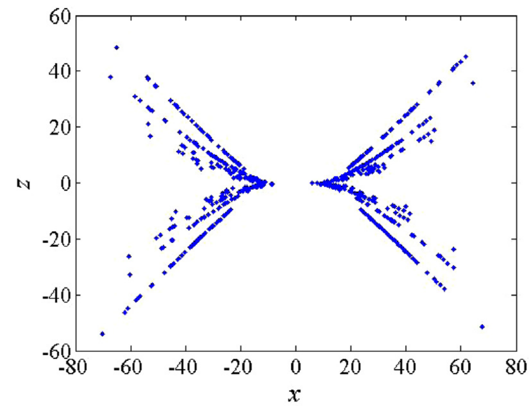

Fig. 2. Poincaré map projected in the $x-z$ plane, when $(a, b, c, d) = (3, 5, 20, 5)$.

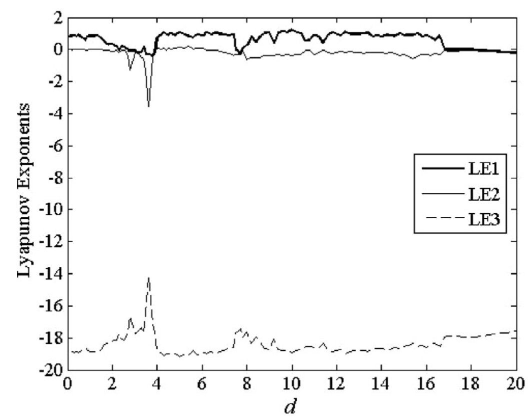

Fig. 3. The Lyapunov exponent spectra versus d ($a = 3$, $b = 5$, $c = 20$).

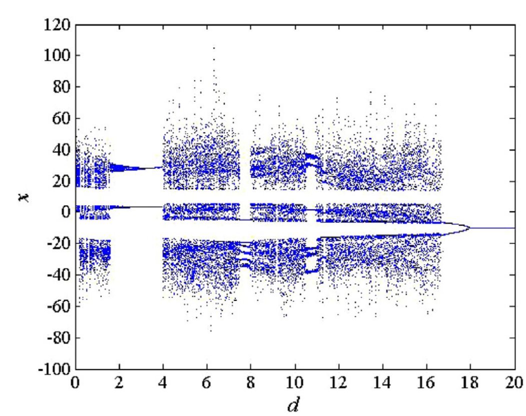

Fig. 4. Bifurcation diagrams of $x(t)$ versus d ($a = 3$, $b = 5$, $c = 20$).

shown in Fig. 1, where the Lyapunov exponents are $LE_1 = 1.0776$, $LE_2 = 0.1416$, $LE_3 = -19.2192$. The Lyapunov dimension of the system is $D_L \approx 2.0634$. That is to say, the system (1) has strange attractors, positive Lyapunov exponents and fractional Lyapunov dimension. The Poincaré map formed in x–z plane of the proposed system is shown in Fig. 2.

For system (1), $\nabla V = \partial\dot{x}/\partial x + \partial\dot{y}/\partial y + \partial\dot{z}/\partial z = -a + b - c$

In order to ensure the dissipation of the system, it is required that

$$(-a + b - c) < 0 \tag{2}$$

Then the system can contract at an exponential rate $e^{(-a+b-c)t}$. When $a = 3$, $b = 5$, $c = 20$, the system converges at a rate of e^{-18t}. Therefore, each volume cell containing the trajectories of the