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Wavelet based spectral analysis of optical solitons

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ABSTRACT

The propagation of solitons or a pulse or a signal through optical fibers has been a major area of research given its potential applicability in all optical communication systems. In a modern optical communication system, the transmission link is composed of optical fibers and amplifiers. This manifests in noise, clutters and distortion when the signal propagates through optical fibers, consequently affecting the capacity and performance of the optical system. The dynamics of solitons has therefore become an active field of research in nonlinear optics for couple of decades. The nonlinear Schrodinger's equation (NLSE) with log law nonlinearity governs the propagation of optical solitons through optical fibers and its dynamics. Most of the studies reveal that the optical solitons have Gaussian wave profile called Gaussons. This entails the use of wavelet techniques for the processing of optical solitons.

Signal processing in optical fiber has two distinct areas of investigations; one, the pulse propagation and the other, signal analysis and synthesis. We, in this work, focus on the later, the signal analysis and synthesis in wavelet spectral framework. For this, new wavelet formalism is proposed for analyzing the Gausson signals transmitted through optical fiber by introducing a nonlinear wavelet-like basis of scaling functions leading to wavelet spectra. The proposed method provides computational algorithm to obtain the Gausson parameters – amplitude and the dispersion, the width and velocity of the traveling soliton, which completely characterize the Gaussian signal. In the end, some of the spectral measures useful for further synthesis of the signal are discussed.

1. Introduction

The nonlinear partial differential equations (NPDE) describe variety of physical patterns in nuclear physics, nonlinear molecular and solid state physics, nonlinear optics etc. The solutions of such NPDEs consist of isolated traveling pulses that are free of interactions and have a shape related to velocity [1]. Such solitary wave solutions are called solitons. In particular, the Korteweg–de Vries equation (KdV), the nonlinear Schrodinger equation (NLSE), the coupled nonlinear Schrodinger equation (CNLSE), and the sine-Gordon equation (SGE) have soliton solutions. The soliton solutions are typically obtained by means of the inverse scattering transform and owe their stability to the integrability of the field equations [2]. The mathematical theory of these equations is a broad and very active field of mathematical research.

Among these, NLSE plays a vital role in various areas of STEM disciplines. It appears in the study of nonlinear optics, plasma physics, mathematical biosciences, quantum mechanics, fluid dynamics and several other disciplines. The main feature of the NLSE is that it supports soliton solution. Solitons are stable nonlinear waves or pulses that transport information through optical fibers over trans-continental and trans-oceanic distances in a matter of few femto-seconds. Solitons have been extensively studied in many and diverse branches of physics such as optics, plasmas, condensed matter physics, fluid mechanics, particle physics and even astrophysics [3]. Over the past two decades, the field of solitons and related nonlinear phenomena has substantially advanced and enriched by research in nonlinear optics [4].

The solitons mainly are of two kinds, viz. spatial solitons and temporal solitons. The spatial solitons correspond to the nonlinear effect that can balance the diffraction. The electromagnetic field due to pulse propagation can change the refractive index of the medium while propagating the pulse, thus creating a structure similar to a graded-index fiber. If the field is also a propagating mode of the guide it has created, then it will remain confined and

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it will propagate without changing its shape. For the spatially confined electromagnetic field, it is possible to send pulses that will not change their shape because the nonlinear effects will balance the dispersion. This corresponds to temporal solitons. In optics, the term soliton is used to refer to any optical field that does not change during propagation because of a delicate balance between nonlinear and linear effects in the medium. An optical soliton is a pulse that travels without distortion due to dispersion or other effects. It is a nonlinear phenomenon caused by self-phase modulation (SPM) which means that the electric field of the wave changes the index of refraction seen by the wave (Kerr effect). SPM causes a red shift at the leading edge of the pulse. Solitons occur when this shift is canceled due to the blue shift at the leading edge of a pulse in a region of anomalous dispersion, resulting in a pulse that maintains its shape in both frequency and time [5].

Optical fibers are used to transport optical signals from source to destination. In optical communication systems, there are many applications in which data is collected from an analog sensor or system and transmitted long distances to the end users. Subsequently, the data would be digitalized and transmitted over radio frequency (RF) or fiber optic communication link. The transmission of RF modulated laser beams through optical fibers and the characterization of information transmitted has been the subject of active research for many years [6]. Over the last couple of decades, the optical communication system has undergone a substantial evolution owing to the impressive progress in the development of optical fibers, optical amplifiers as well as transmitters and receivers. Significant improvements have been made during this period to address the problems of absorption, dispersion and non-linear effects in optical fibers. One very powerful way to make a system transparent to fiber impairments is to encode amplitude and phase information which will be immune to the negative effects of dispersion and non-linear interactions [7].

Problems of signal processing in optical fiber are of two distinct categories: one, the pulse propagation and the other, signal analysis. We, in this work, focus on the analysis and synthesis of the signal transmitting through optical fiber. Most of the studies reveal that the optical solitons have Gaussian wave profile. This entails the use of wavelet techniques for the analysis and synthesis of optical signals.

In wavelet inspired approach, the sets of wavelets are employed to approximate signals because of their Gaussian form. The elegance of the wavelet analysis lies in its predominant property of self-similarity that makes wavelet as a powerful tool for analyzing fractal like patterns. The soliton solutions that arise from nonlinear partial differential equations display very strong interaction between the initial conditions and the dynamics involving multiple scales, being able to produce self-similar or fractal-like patterns. Moreover, since the soliton-like solutions have infinite space–time extension, it requires rather appropriate compactly supported basis functions to investigate such structures than the traditional nonlinear tools (inverse scattering, group symmetry, functional transforms). The wavelet technique therefore is the robust methods for the processing of optical solitons. Besides, the wavelet transform enjoys the linear superposition principle, the computational efficiency, and the signal/noise ratio enhancement for a non-sinusoidal and non-stationary signal [8].

In our earlier works, we studied extensively the wavelet interaction with solitons arising as the solutions of non-linear partial differential equations, viz. non-linear Schrodinger equation, Sine–Gordon equation, Korteweg–de Vries equation [9,10]. Also, we explored the strong relationships existing between wavelets, solitons and probability distributions [11] and conducted spectral analysis in stochastic framework. Moreover, we extended the wavelet methods to random processes and applied the technique for the analysis of genomic sequences [12], and neural networks [13].

We in this work proposed a wavelet formulation suitable for analyzing the optical soliton signals by introducing a nonlinear wavelet-like basis of scaling functions made by localized analytical nonlinear solutions. The Gaussian parameters – amplitude and the dispersion– can completely characterize the signal, the method provides a computational procedure to obtain the amplitude, width and velocity of the optical soliton arising as the traveling solutions of certain nonlinear equations.

2. Governing equation

Among the NLPDEs representing non-linear phenomena, nonlinear Schrodinger's equation (NLSE) is one of the most celebrated NLPDEs that govern the propagation of solitons through optical fibers. There are several forms of the NLSE depending on the nonlinearity and perturbation being considered. The nonlinearities being considered in the study of NLSE are Cubic/Karr law nonlinearity, power law nonlinearity and log law nonlinearity. Especially, the NLSE with logarithmic nonlinearity possesses soliton-like solutions in any number of dimensions. Due to their Gaussian shape, these soliton solutions are known as Gaussons. The solitons are the outcome of a delicate balance between dispersion and nonlinearity. Biswas and Konar [14] employed the variational principle to study chirped solitons that propagate through optical fibers and is governed by the dispersion-managed NLSE and extended it to obtain the adiabatic evolution of soliton parameters in the presence of perturbation terms for such fibers considering both Gaussian and super-Gaussian solitons. Khalique and Biswas [15] in their work integrated the nonlinear Schrodinger equation with log-law nonlinearity by the aid of Lie symmetry approach and obtained stationary solutions that exhibit Gaussian features. We present the review of some of the different forms of NLSEs and their solutions.

The dimensionless NLSE non-Kerr media is given as

$$iq_t + aq_{xx} + bF(|q|^2)q = 0 \tag{1}$$

where q is the dependant variable of x and t; x and t represent the spatial and the temporal variables respectively. The constant parameters a and b respectively represent the coefficients of group velocity dispersion (GVD) and nonlinearity. The first term is the temporal evolution of pulses as they propagate down the optical fiber, the second term is the GVD term and the third term is the nonlinear term where the function F dictates the type of non-linearity. The governing Eq. (1) has a soliton solution which is of the form

$$q(x,t) = g(s)e^{i\phi}, \quad s = x - vt, \quad \phi = -\kappa x + \omega t + \theta \tag{2}$$

where $g(s)$ represents the shape of the soliton/pulse described by NLSE, while $\phi(x, t)$ is phase of the soliton, v is the velocity of the soliton, κ is the frequency, ω is the soliton wave number and θ is the pulse constant.

With the perturbed term R suitably chosen to describe soliton perturbation, the NLSE takes the form

$$iq_t + aq_{xx} + b\log(|q|^2)q = i\varepsilon R \tag{3}$$

where parameter ε is related to the relative width of the spectrum arises due to quasi-monochromicity.

The most recent generalization of NLSE is the Biswas–Milovic equation (BME) with log law nonlinearity that incorporates imperfections arising in the context of optical solitons, which is given as

$$i(q^m)_t + a(q_m)_{xx} + bF(|q|^2)q^m = 0 \tag{4}$$

where m, $m > 1$, is the exponential parameter. For $m = 1$, BME reduces to NLSE. The fiber imperfections and its geometry lead to