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Dark optical solitons with power law nonlinearity using G'/G-expansion

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ABSTRACT

This paper studies the dynamics of dark optical solitons. The G'/G-expansion approach is utilized. The byproduct of this approach is the singular periodic solution of the governing nonlinear Schrödinger's equation for its corresponding parameter regime. The constraint conditions are also in place for the existence of dark solitons.

1. Introduction

The dynamics of soliton propagation through optical fibers for trans-continental and trans-oceanic distances is one of the most attractive areas of research in physical and engineering sciences. There are several types of nonlinearity that are studied in this context. While Kerr law nonlinearity is the most popular, there are several lesser-known nonlinear media that are studied in various areas. One such nonlinearity is the power law that generalizes Kerr law. This paper will therefore address the nonlinear Schrödinger's equation (NLSE) with power law nonlinearity in presence of perturbation terms that will be considered with full nonlinearity.

The integrability aspect will be the focus of this paper. The aim is to extract dark soliton solution to the perturbed NLSE with full nonlinearity. While there are several integration tools that are available to solve such problems, this paper will address the G'/G-expansion approach. This is indeed a powerful as well as popular integration architecture that has gained fame in the past few years [1–9]. This tool will lead to the dark 1-soliton solution of NLSE under certain parameter regime that will be listed as a constraint condition. This NLSE with Hamiltonian perturbation terms was integrated in the past by ansatz approach [10].

2. Review of the G'/G-expansion

In this section, we describe the G'/G-expansion method [1–9] for finding traveling wave solutions of nonlinear evolution equations (NLEEs) and then subsequently it will be applied to solve the NLSE with power law nonlinearity [10].

We assume that the given NLPDE for $u(x,t)$ is in the form

$$P(u, u_t, u_x, u_{xx}, u_{xt}, u_{tt}, \ldots) = 0, \tag{1}$$

where P is a polynomial. The essence of the G'/G-expansion method can be presented in the following steps:

Step 1: To find the traveling wave solutions of Eq. (1), we introduce the wave variable

$$u(x, t) = U(\xi), \quad \xi = x - ct. \tag{2}$$

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Substituting Eq. (2) into Eq. (1), we obtain the following ODE

$$Q\left(U, U', U'', \ldots\right) = 0. \tag{3}$$

Step 2: Eq. (3) is then integrated as long as all terms contain derivatives where integration constants are considered zeros.
Step 3: Introduce the solution $U(\xi)$ of Eq. (3) in the finite series form

$$U(\xi) = \sum_{l=0}^{N} a_l \left(\frac{G'(\xi)}{G(\xi)}\right)^l \tag{4}$$

where a_l are real constants with $a_N \neq 0$ and N is a positive integer to be determined. The function $G(\xi)$ is the solution of the auxiliary linear ordinary differential equation

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \tag{5}$$

where λ and μ are real constants to be determined.
Step 4: Determining N, can be accomplished by balancing the linear term of highest order derivatives with the highest order nonlinear term in Eq. (3).
Step 5: Substituting the general solution of (5) together with (4) into Eq. (3) yields an algebraic equation involving powers of G'/G. Equating the coefficients of each power of G'/G to zero gives a system of algebraic equations for a_l, λ, μ and c. Then, we solve the system with the aid of a computer algebra system, such as *Maple*, to determine these constants. Next, depending on the sign of the discriminant $\Delta = \lambda^2 - 4\mu$, we get solutions of Eq. (3). So, we can obtain exact solutions of the given Eq. (1).

3. Application to NLSE with power law nonlinearity

Let us demonstrate the application of the G'/G-expansion method for finding the new exact traveling wave solutions of perturbed NLSE with power law nonlinearity in the following form [10]

$$iq_t + aq_{xx} + b|q|^{2m}q = icq_x - i\gamma q_{xxx} + is(|q|^{2m}q)_x + ir(|q|^{2m})_x q, \tag{6}$$

where a, b, c, γ, s and r are all real valued constants. Also, the exponent m represents the power law nonlinearity parameter. For the perturbation terms on the right hand side α represents the inter-modal dispersion, γ is the coefficient of third order dispersion, s is the coefficient of self-steepening term while r is the coefficient of nonlinear dispersion. The self-steepening and nonlinear dispersion terms are considered with full nonlinearity, namely their intensities are considered with an exponent m, in order to maintain the problem on a generalized setting.

We assume Eq. (6) has the traveling wave solution of the form

$$q(x,t) = U(\xi)e^{i(\alpha x + \beta t)}, \quad \xi = i(kx - \omega t), \tag{7}$$

where α, β, k and ω are constants, all of them are to be determined. Thus, from the wave transformation (7), we have

$$q_t = i\left(\beta U - \omega U'\right)e^{i(\alpha x+\beta t)}, \tag{8}$$

$$q_x = i\left(\alpha U + kU'\right)e^{i(\alpha x+\beta t)}, \tag{9}$$

$$q_{xx} = -(\alpha^2 U + 2\alpha k U' + k^2 U'')e^{i(\alpha x+\beta t)}, \tag{10}$$

$$q_{xxx} = -i(\alpha^3 U + 3\alpha^2 k U' + 3\alpha k^2 U'' + k^3 U''')e^{i(\alpha x+\beta t)}, \tag{11}$$

$$(|q|^{2m}q)_x = i(\alpha U^{2m+1} + k(U^{2m+1})')e^{i(\alpha x+\beta t)}, \tag{12}$$

and

$$(|q|^{2m})_x q = ik(U^{2m})' U e^{i(\alpha x+\beta t)}. \tag{13}$$

Inserting the expressions (8)–(13) into Eq. (6), we obtain nonlinear ODE in the form

$$(c\alpha + \gamma\alpha^3 - \beta - a\alpha^2)U + (\omega - 2a\alpha k + ck + 3\alpha^2 k\gamma)U' + (3\alpha k^2\gamma - ak^2)U'' + (b + s\alpha)U^{2m+1} + k^3\gamma U''' + sk(U^{2m+1})' + rk(U^{2m})'U = 0. \tag{14}$$

Balancing U''' with $U'U^{2m}$ in Eq. (14) give

$$N + 3 = N + 1 + 2mN \Leftrightarrow 3 = 2mN + 1 \Leftrightarrow N = \frac{1}{m}.$$

We then assume that Eq. (14) has the following formal solutions:

$$U(\xi) = A\left(\frac{G'}{G}\right)^{1/m}, \quad A \neq 0 \tag{15}$$

where A is a constant to be determined later and G satisfies Eq. (5). From Eq. (15), we obtain

$$U' = -\frac{1}{m}A\left(\frac{G'}{G}\right)^{(1/m)+1} - \frac{1}{m}A\lambda\left(\frac{G'}{G}\right)^{(1/m)} - \frac{1}{m}A\mu\left(\frac{G'}{G}\right)^{(1/m)-1}. \tag{16}$$