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# Effects of multi-Schell beams on orbital angular momentum of entangled signal photons in low-order turbulence aberration channels

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ABSTRACT

We model the detection and crosstalk probability of orbital angular momentum (OAM) states of the entangled signal photon in the Kolmogorov channels of the low-order turbulence aberrations and by the Rytov approximation. The results show that lower OAM mode number of signal photons and larger sub-beam number of multi-Gaussian Schell-model pump beam, the less susceptible of the detection probability of the signal photon to spatial coherence of source and turbulence aberrations is achieved. The maximum crosstalk probability is decrease as the decreasing of the sub-beam number of multi-Gaussian Schell-model. Enlarging OAM difference value or decreasing sub-beam number of multi-Gaussian Schell-model pump beam results in a lower crosstalk probability of the OAM of entangled signal photons.



## 1. Introduction

In recent years, the interest in entangled orbital angular momentum (OAM) states in atmospheric turbulence is steadily growing for its promise to realize new types of quantum communication protocols. Based on the two-photon wavefunction, the effects of turbulence on entangled photons were studied and indicated that entangled states with smaller waists and larger OAM quantum numbers will be more robust to turbulence [1]. The robustness of OAM-encoded quits in atmospheric turbulence were studied and tested; the entangled photons are less robust to the effects of Kolmogorov turbulence compared to single photons, and signal photons are more robust than single photons in the lowest-order mode [2]; the quantum channel capacity of OAM entanglement of two photons is surprisingly robust to the atmospheric turbulence in experiment [3]; the information content of quits encoded in a bidimensional subspace of the OAM degree of freedom of photons was tested [4]. The entanglement of entangled photon pair decreases with the increase of the propagation distance z and the turbulence intensity in the atmosphere channel and in the same intensity of atmospheric turbulence of the atmospheric channel, the bigger the orbital angular momentum index, the slower the entanglement decline and the further the propagation distances [5]. An infinitesimal propagation equation that models the spatial evolution of a density operator for an OAM entangled multiphoton state propagating through turbulent atmosphere was derived [6,7]. According to the work of Noll [8], the tilt aberration is the main component of the turbulence aberration induced upon a wavefront, which is passing through the atmosphere. Therefore, the effects of low-order Zernike turbulence aberrations including tilt aberration on OAM entanglement states in a weak fluctuation region, based on Zernike-model expansion of turbulence phase aberrations and non-Kolmogorov spectrum model of index-of-refraction fluctuation, was analyzed by Sheng et al. [9]. Based on the paraxial approximation of Laguerre–Gaussian beam propagation in communication system, non-Kolmogorov spectrum model of index-of-refraction fluctuation in slant path, the Zernike polynomial expansion of atmospheric turbulence-aberration turbulence-channel, the residue aberration effect of Zernike tilt correction on entangled OAM states was analyzed [10]. It is well known that partially coherent beams are less influenced by turbulent atmosphere than completely coherent beams [11,12], but it was shown that the signal photon detection probability of entangled photon pairs decay nonlinearly with space coherence decreasing of beams [13]. We also note that with the proper choice of the source parameters of multi-Gaussian Schell-beam the flattened intensity profile can still be constructed before the turbulence atmosphere starts affecting the field [14]. However, to our knowledge, the theory of OAM entanglement states produced by down-conversion using multi-Gaussian Schell-model pump for

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light propagation in low-order turbulence aberration channels is less developed.

In this paper, the objective is to characterize the performance of OAM entangled states by the signal photon detection probability when the entangled state is generated by the multi-Gaussian Schell-model pump beam and propagation in the low-order Kolmogorov turbulence atmosphere.

## 2. Detection probability of entangled signal photon of multi-Gaussian Schell-model pump

In the case of multi-Gaussian Schell-model pump and non-collinear phase matching of signal and idler photons, the detection probability $P(l_1)$ of entangled signal photon (DPESP) in turbulent atmosphere channel is given by [2]

$$P(l_1) = \int\int \langle \Phi(r,\varphi)\Phi^*(r,\varphi)\rangle_s R_{l_1}(r,z) R^*_{l_1}(r,z) \exp\left[-\tfrac{1}{2}D_S(r,\Delta\varphi)\right] \times \exp[\mathrm{i}\Delta l' \Delta\varphi] r\mathrm{d}r\mathrm{d}\Delta\varphi \tag{1}$$

where $\langle \Phi(r,\varphi)\Phi^*(r,\varphi)\rangle_s$ is the elements of the cross-spectral density matrix of pump beam, $^*$ denotes complex conjugate, $\Phi(r,\varphi)$ is the transverse spatial profile of the pump beam at the input face ($z=0$) of the crystal and $\langle\cdot\rangle$ represents an ensemble average for the source. $D_S(r,\Delta\varphi)$ is the phase structure function of the aberration, $S$ is the turbulence phase aberration [8] that depends on the prosperities of the turbulence, $a_1$ is a piston which indicates a uniform shift in the entire wavefront, $S_{tilt}$ is a Z-tilt turbulence aberration, $S_{defo}$ is a turbulence defocus aberration, $S_{asti}$ is a astigmatism turbulence aberration and $S_{coma}$ is a coma turbulence aberration. $\Delta\varphi=\varphi'-\varphi$, $\Delta l'=l_1-l_p$, $l_p$ is the initial OAM quantum number of a pump mode and $l_1$ is OAM quantum number of a signal mode. $R_{l_1}(r)$ is a radial mode

$$R_{l_1}(r,z) = \frac{1}{w}\sqrt{\frac{1}{(|l_1|)!}}\left(\frac{r}{w}\right)^{|l_1|}\exp\left(-\frac{r^2}{2w^2}\right)\exp\left(-\frac{\mathrm{i}kr^2}{4R}\right) \tag{2}$$

where $w_i = w_0\sqrt{1+(z/z_R)^2}$ is the spot size, $R_i=z[1+(z_R/z)^2]$ is the radius of wavefront curvature, $z_R=(1/2)kw_0^2$ is the Rayleigh range, $k=2\pi/\lambda$ is the wavelength and $w_0$ is a beam width parameter at the beam waist at launch.

For multi-Gaussian Schell-model pump beams with OAM quantum number $l_p=l_1+l_2=0$, $l_2$ is the initial OAM quantum number of an idler photon mode, the elements of the cross-spectral density matrix of pump beams can be expressed in the form [14]

$$\langle \Phi(\mathbf{r},\varphi)\Phi^*(\mathbf{r}',\varphi')\rangle_s = \frac{A_p}{C_0}\exp\left[-\frac{r^2+r'^2}{4w_p^2}\right]\sum_{m=1}^{M}\binom{M}{m}\frac{(-1)^{m-1}}{m} \times \exp\left[-\frac{(\mathbf{r}-\mathbf{r}')^2}{2m\rho_{s0}^2}\right] \tag{3}$$

where $A_p$ is a constant, $w_p$ is the pump beam waist width, $\rho_{s0}$ is the transverse coherence width, $C_0=\sum_{m=1}^{M}\frac{(-1)^{m-1}}{m}\binom{M}{m}$ is the normalization factor, $\binom{M}{m}$ stand for binomial coefficients, $M$ is the number of the multi-Gaussian Schell-model pump beam.

Then, we can substitute Eqs. (2) and (3) back into Eq. (1) to get the DPESP $P=(l_1)$ produced by down-conversion using multi-Gaussian Schell-model pump beam

$$P(l_1) = \frac{C}{w^2 C_0(|l_1|)!}\sum_{m=1}^{M}\binom{M}{m}\frac{(-1)^{m-1}}{m}\int\int\left(\frac{r}{w}\right)^{2|l_1|} \times \exp[\mathrm{i}\Delta l'\Delta\varphi] r\mathrm{d}r\mathrm{d}\Delta\varphi \exp\left[-\left(\frac{2\sin^2(\Delta\varphi/2)}{m\rho_{s0}^2}+\frac{1}{2w_p^2}+\frac{1}{w^2}\right)r^2 - \frac{1}{2}D_S(r,\Delta\varphi)\right] \tag{4}$$

## 3. Effects of turbulent aberrations

### 3.1. Turbulent channel

For the Kolmogorov channel, the spectrum of atmospheric turbulence is represented as [9]

$$\phi_n = 0.033C_n^2(z)\kappa^{-11/3} \tag{5}$$

where, $C_n^2(z)$ is the refractive-index structure parameter, which is altitude dependent and is given by

$$C_n^2(h) = 0.00594(\left(\frac{\upsilon}{27}\right)^2 (h\times 10^{-5})^{10}\exp\left(-\frac{h}{1000}\right) + 2.7\times 10^{-16} \times \exp\left(-\frac{h}{1500}\right) + C_n^2(0)\exp\left(-\frac{h}{100}\right) \tag{6}$$

where $h=z\cos\theta$ is altitude, $\upsilon=21$ m/s is the rms wind speed, $C_n^2(0)$ is the structure parameter at the ground and $\theta$ is the zenith angle of communication channel.

### 3.2. Z-tilt aberration

For the Z-tilt turbulence aberration [9] $S_{tilt}=2a_2 r\cos\varphi + 2a_3 r\sin\varphi$, where $a_{2,3}$ are the coefficients of the corresponding Zernike polynomials, the Z-tilt aberration structure function is given by

$$D_{tilt}\left(\left|2r\sin\left(\frac{\Delta\varphi}{2}\right)\right|\right) = 3.59\left(\frac{D}{r_0}\right)^{5/3} r^2[1-\cos(\Delta\varphi)] \tag{7}$$

where $D$ is the diameter of circular sampling aperture and Fried's coherence diameter in Kolmogorov channel is given by

$$r_0 = 0.185\left[\frac{k^2\int_0^z C_n^2(\xi)(1-\xi/z)^{5/3}d\xi}{4\pi^2}\right]^{-3/5} \tag{8}$$

The DPESP in Z-tilt turbulence aberration channel becomes

$$P(l_1) = \frac{C}{w^2 C_0(|l_1|)!}\sum_{m=1}^{M}\binom{M}{m}\frac{(-1)^{m-1}}{m}\int\int\left(\frac{r}{w}\right)^{2|l_1|} \times \exp[\mathrm{i}\Delta l'\Delta\varphi] r\mathrm{d}r\mathrm{d}\Delta\varphi \exp\left[-\left(\frac{2\sin^2(\Delta\varphi/2)}{m\rho_{s0}^2}+\frac{1}{2w_p^2}+\frac{1}{w^2} + 3.59\left(\frac{D}{r_0}\right)^{5/3}\left[\sin\left(\frac{\Delta\varphi}{2}\right)\right]^2\right)r^2\right] \tag{9}$$

### 3.3. Defocus aberration

For the defocus turbulence aberration [9] $S_{defo}(r)=a_4\sqrt{3}(2r^2-1)$, where $a_4$ is the coefficient of the corresponding Zernike polynomials, the defocus aberration structure function $D_{defo}(|2r\sin(\Delta\varphi/2)|)\approx 0$.