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Geometric phase and non-stationary state

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ABSTRACT

By investigating a particle motion in a three-dimensional potential barrier with moving boundary, we find that due to an alteration of boundary conditions, the wave function pick up an additional nonlocal phase factor independent on the dynamics of physical system. By compare the nonlocal phase with the geometric phase of the physical system, furthermore, we find that the nonlocal feature of quantum behavior can fully be described by its geometric phase.

1. Introduction

For a stationary quantum state, the wave function is only a property of a statistical ensemble of similarly prepared systems and tells nothing about the time evolution properties of individual physical systems in terms of the probability interpretation. Thus, the state vector from the physical system with a constant Hamiltonian in the standard quantum mechanics is time-independent and does not mean that individual system does not depend on time. It is clear that, however, the observed world depends on the time [1,2]. Therefore, it is necessary to reconcile an observed time-dependence with a time-independent wave function of the universe.

It is well-known the time-dependent quantum Hamiltonian has been investigated for past several decades. Such a system has also been shown to be a versatile tool for testing many concepts and approaches used in quantum theory [3–7]. A serial of new important discoveries or new applications has been obtained about the time-dependent quantum systems. Such as the path integral method [8], the coherent and squeezed states [9,10], quantum tunneling in Josephson contacts and SQUIDs [11], the invariant theory of time-dependent quantum system [12], the Zeno effect [13], the geometric phase [14–23], and quantum traps and cavity QED [24].

Nowadays, a lot of time-dependent phenomena is witnessed in quantum optics, quantum information theory, condensed matter physics and particle physics, such as the big bang theory [25], neutrino oscillations [26] and time-dependent tunneling [27]. Such phenomena reveal the facts that quantum non-stationary systems have continually been a living and very interesting subject of quantum physics from many different areas up to today.

Another interesting study is to observe the time-dependent processes in mesoscopic quantum devices [28]. In the paper, we investigate a particle motion confined in a three-dimensional infinite square well with a time-dependent boundary in which appears to be modeled by a quantum device. The result shows that differently from a stationary boundary, the wave function picks up an additional nonlocal phase factor. By comparing the phase factor with the geometric phase, we find that the nonlocal feature of quantum behavior may fully be described in terms of the geometric phase.

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2. Quantum system with a moving three-dimensional boundary

Let us consider a particle with mass m confined in a three-dimensional infinite square well with a time-dependent boundary. The time-dependent Schrödinger equation is given by

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = H(t)|\psi(t)\rangle, \tag{1}$$

where the Hamiltonian operator $H(t)$ is expressed by a sphere coordinate, i.e.,

$$H(t) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + V(r,t), \tag{2}$$

where θ and φ are azimuthal angles and r is radius coordinate, while the potential energy function $V(r, t)$ is defined to be zero if $0 \le r \le a(t)$, and infinite otherwise, i.e.,

$$V(r,t) = \begin{cases} 0, & 0 \le r \le a(t) \\ \infty, & r < 0, \text{or}, r > a(t) \end{cases}, \tag{3}$$

which can be simulated in the sense by a quantum device.

Suppose the potential boundary moves linearly at a constant rate β in terms of $a(t) = a_0 + \beta t$ with $a_0 = a(0)$. Thus the wall speed parameter β is positive for expanding and negative for contracting wells. An exact solution to the Schrödinger equation (1) satisfying the time-dependent boundary conditions may be written as

$$|\Psi(r, a(t))\rangle = \sum_{n_r} b_{\mathrm{ln}_r} |\Psi_{\mathrm{ln}_r m_l}(r, a(t))\rangle, \tag{4}$$

where the coefficients b_{ln_r} are time-independent and determined by the initial condition, while

$$|\Psi_{\mathrm{ln}_r m_l}(r, a(t))\rangle = e^{i\alpha_{\mathrm{ln}_r}(r,t)}\phi_{\mathrm{ln}_r m_l}(r, a(t))e^{-i/\hbar\int_0^t E_{\mathrm{ln}_r}dt}, \tag{5}$$

where

$$\phi_{\mathrm{ln}_r m_l} = \left[\frac{-2}{a^3 j_{l-1}(\chi_{\mathrm{ln}_r})j_{l+1}(\chi_{\mathrm{ln}_r})}\right]^{1/2} j_l\left(\frac{\chi_{\mathrm{ln}_r}r}{a(t)}\right)Y_{lm_l}, \tag{6}$$

with

$$Y_{lm_l}(\theta,\varphi) = N_{lm_l}P_l^{|m_l|}(\cos\theta)e^{im_l\varphi}, \tag{7}$$

where $P_l^{|m_l|}$ is an associated Legendre function.

It is obvious that $\phi_{\mathrm{ln}_r m_l}$ is an eigenstate of Hamiltonian (2), i.e., $H\phi_{\mathrm{ln}_r m_l} = E_{\mathrm{ln}_r}\phi_{\mathrm{ln}_r m_l}$ with the time-dependent eigenvalue $E_{\mathrm{ln}_r}(t) = (\hbar^2/(2ma^2(t)))\chi^2_{\mathrm{ln}_r}(n_r = 1, 2, 3\ldots, l = 0, 1, 2, \ldots,$ and $m_l = l, l-1, \ldots, -l)$.

Inserting Eq. (5) into Eq. (1), we find

$$\left(\frac{\partial\alpha_{\mathrm{ln}_r}}{\partial r}\right)^2 = -\frac{2m}{\hbar}\frac{\partial a}{\partial t}, \tag{8}$$

and

$$-\frac{\hbar}{2m}\left[2\frac{\partial\alpha_{\mathrm{ln}_r}}{\partial r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\alpha_{\mathrm{ln}_r}}{\partial r}\right)\right]\phi_{\mathrm{ln}_r m} = \frac{\partial\phi_{\mathrm{ln}_r m}}{\partial t}. \tag{9}$$

Substituting Eq. (6) into Eq. (9), one has

$$\frac{\partial^2\alpha_{\mathrm{ln}_r}}{\partial r^2}j_l\left(\frac{\chi_{\mathrm{ln}_r}r}{a}\right) + \frac{\partial\alpha_{\mathrm{ln}_r}}{\partial r}\left\{\frac{2}{r}j_l\left(\frac{\chi_{\mathrm{ln}_r}r}{a}\right) + \frac{2}{2l+1}\frac{\chi_{\mathrm{ln}_r}}{a}\left[lj_{l-1}\left(\frac{\chi_{\mathrm{ln}_r}r}{a}\right) - (l+1)j_{l+1}\left(\frac{\chi_{\mathrm{ln}_r}r}{a}\right)\right]\right\} - \frac{2m}{\hbar}\frac{\partial a}{\partial t}\left\{\frac{3}{2a}j_l\left(\frac{\chi_{\mathrm{ln}_r}r}{a}\right) + \frac{1}{2l+1}\frac{\chi_{\mathrm{ln}_r}r}{a^2}\left[lj_{l-1}\left(\frac{\chi_{\mathrm{ln}_r}r}{a}\right) - (l+1)j_{l+1}\left(\frac{\chi_{\mathrm{ln}_r}r}{a}\right)\right]\right\} = 0, \tag{10}$$

which leads to an exact solution, we find

$$\alpha_{\mathrm{ln}_r} = \frac{m}{2\hbar}\frac{\partial a}{\partial t}\frac{r^2}{a}, \tag{11}$$

under the condition,

$$a(t) = a_0 + \beta t. \tag{12}$$

From Eq. (11), we see that the phase factor α_{ln_r} is not relation to quantum number and depends only on the alteration of boundary conditions. Therefore the phase factor is independent of the dynamics of the physical system because there do not exist any interaction or force. Thus, the phase factor is nonlocal and therefore α_{ln_r} is called as the nonlocal phase.

Thus the wave function (5) may be exactly expressed as

$$|\Psi_{\mathrm{ln}_r m_l}(r, a(t))\rangle = e^{i(m/2\hbar)(\partial a/\partial t)(r^2/a)}\left(\frac{-2}{a^3 j_{l-1}(\chi_{\mathrm{ln}_r})j_{l+1}(\chi_{\mathrm{ln}_r})}\right)^{1/2} \times j_l\left(\frac{\chi_{\mathrm{ln}_r}r}{a(t)}\right)Y_{lm_l}(\theta,\varphi)\times e^{-i/\hbar\int_0^t E_{\mathrm{ln}_r}(t)\,dt}. \tag{13}$$

From Eq. (13), we see that a nonlocal phase factor is emerged from the alteration of moving boundary. The space-dependent part of the phase ensures that the particle moves in concert with the packet and stays between the nodes. Especially, this exact solution is valid for both fast or slow expansions and contractions.

The results imply that the wave function of a particle, described by the time-dependent Schrödinger equation, is initially and finally free may pick up a phase factor by alteration of boundary conditions. This alters the observable properties of the system even though the particle has not come near the conning walls.

3. Guidance of the wave

According to the Bohm theory [1–3], an individual physical system comprises not only a wave propagating in space and time together with a point particle which moves continuously under the guidance of the wave mathematically described by the Schrödinger equation but also an extra information about the particle motion. The guidance formula may be expressed by

$$\dot{r}(t) = \frac{1}{m}\nabla S(r,t)|_{r=r(t)}, \tag{14}$$

where $S(r, t)$ is an action of physical system from the whole phase of wave function (13), which may be written as

$$S(r,t) = \frac{m}{2}\frac{\partial a}{\partial t}\frac{r^2}{a} - \int_0^t E_{\mathrm{ln}_r}(t)dt + m_l\hbar\varphi. \tag{15}$$

Using Eqs. (14) and (15), the particle moving trajectory may be obtained by

$$r(t) = r_0\left(1 + \frac{\beta t}{a_0}\right), \tag{16}$$

and

$$\varphi - \varphi_0 = \frac{m_l\hbar}{\beta m\sin\theta}\left(\frac{1}{a_0} - \frac{1}{a}\right), \tag{17}$$

where the time-independent initial condition $r(0) = r_0$ is used so that $\dot{r}(t) = \beta r_0/a_0 =$ constant. The speed varies linearly with the