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Optical soliton perturbation with spatio-temporal dispersion in parabolic and dual-power law media by semi-inverse variational principle

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ABSTRACT

This paper studies the perturbed optical solitons with parabolic and dual-power law nonlinearities in presence of spatio-temporal dispersion. The semi-inverse variational principle is applied to extract an analytical 1-soliton solution to the governing equation. There are constraint conditions that naturally fall out for the existence of these solitons.

1. Introduction

The dynamics of optical solitons is governed by the nonlinear Schrödinger's equation (NLSE) [1–30]. These solitons form the fabric of information carrier through various kinds of optical fibers across oceans and continents in a matter of a few femto-seconds. Besides the engineering aspects of solitons and optical fibers, it is also important to focus on the integrability aspects of these soliton dynamics. There are several tools of integrability that are available to study this aspect. Some of the commonly studied tools are the traveling wave hypothesis, ansatz approach, Lie symmetry approach, exp-function method, G'/G-expansion method, tanh–coth approach, collective variables method

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and several others. While these methods reveal exact soliton solutions or obtains the parameter dynamics of the solitons, there exists another method that obtains the analytical soliton solution. This is the semi-inverse variational principle (SVP). This integration technique obtains an analytical soliton solution although not necessarily exact. Such solutions are immensely helpful in the community of optical solitons.

2. Governing equation

The NLSE with spatio-temporal dispersion (STD) in presence of perturbation terms with full nonlinearity is given by [11]

$$iq_t + aq_{xt} + bq_{xx} + F(|q|^2)q = i[\alpha q_x - \gamma q_{xxx} - i\sigma q_{xxxx} + \lambda(|q|^{2m}q)_x + \nu(|q|^{2m})_x q] \tag{1}$$

In (1), the first term represents linear evolution of the pulses, while a and b, respectively, represent the coefficients of STD and group-velocity dispersions (GVD). The nonlinear functional F on the left side represents parabolic or dual-power law nonlinearity. Also, $F(|q|^2)q$ is k times continuously differentiable, so that

$$F(|q|^2)q \in \bigcup_{m,n=1}^{\infty} C^k((-n, n) \times (-m, m); R^2). \tag{2}$$

The independent variables are x and t which represent spatial and temporal variables, respectively, while the dependent variable is $q(x, t)$ which is the complex-valued wave profile.

From the perturbation terms on the right side, α is the inter-modal dispersion, γ represents the third order dispersion, σ is the fourth order dispersion, λ is the self-steepening term and ν is the coefficient of nonlinear dispersion. Finally, the parameter m is the full nonlinearity parameter. The inclusion of higher order dispersion (HODs) term is out of necessity. If the GVD is negligible, it is these HODs that make up for the low GVD. The self-steepening term is considered to avoid shock wave formation. The inclusion of the STD term was proposed during 2012 since STD makes the NLSE well-posed [15,19].

2.1. Semi-inverse variational principle

In order to apply the SVP to (1), the starting hypothesis is the traveling wave solution that is given by [11]

$$q(x, t) = g(s)e^{i\phi}, \tag{3}$$

where $g(s)$ represents the shape of the wave profile and

$$s = x - vt, \tag{4}$$

with v being the soliton speed. The phase component $\phi(x, t)$ is defined as

$$\phi = -\kappa x + \omega t + \theta \tag{5}$$

where κ represents the soliton frequency and ω is the soliton wave number while θ represents the phase constant. Substituting this hypothesis into (1) and decomposing into real and imaginary parts yield the following two relations

$$\sigma g^{(iv)} - P_2 g'' - P_1 g - F(g^2)g + \lambda\kappa g^{2m+1} = 0 \tag{6}$$

and

$$(v - a\omega - av\kappa + 2b\kappa + \alpha + 3\gamma\kappa^2 + 4\sigma\kappa^3)g' - (\gamma + 4\sigma\kappa)g''' + \{(2m+1)\lambda + 2m\nu\}g^{2m}g' = 0 \tag{7}$$

respectively, where

$$P_1 = -\omega + a\omega\kappa - b\kappa^2 - \alpha\kappa - \gamma\kappa^3 - \sigma\kappa^4 \tag{8}$$

and

$$P_2 = -av + b + 3\gamma\kappa + 6\sigma\kappa^2 \tag{9}$$

The notations $g' = dg/ds$, $g'' = d^2g/ds^2$ and so on, are adopted in (6) and (7). From the imaginary part equation (7), setting the coefficients of the linearly independent functions to zero yield the constraint conditions

$$\gamma + 4\sigma\kappa = 0 \tag{10}$$

and

$$(2m+1)\lambda + 2m\nu = 0 \tag{11}$$

while the soliton speed falls out to be

$$v = \frac{a\omega - 2b\kappa - \alpha + 8\sigma\kappa^3}{1 - a\kappa} \tag{12}$$

This shows that soliton speed will remain the same irrespective of the type of nonlinearity under study. Now, multiplying both sides of the real part equation (6) by g' leads to

$$\sigma(g'')^2 - 2\sigma g' g''' + P_1 g^2 + P_2 (g')^2 - \frac{\lambda\kappa g^{2m+2}}{m+1} + 2\int^g F(h^2)h\,dh = K \tag{13}$$