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Degeneracy from quadric cones for projective reconstruction from two views

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ABSTRACT

Fundamental matrix encapsulates all the geometric information in two views, and it plays an important role in many applications of three-dimensional computer vision. However, some configurations have an inherent ambiguity, namely, degeneracy, and no matter how many matched image points are used, the fundamental matrix could not be determined uniquely. It is well-known that degeneracies occur when all the space points and both the camera centers belong to a ruled quadric. But it is not so well-known how great the degenerate degrees in different configurations are. In this paper, we discuss the degeneracies caused by a quadric cone and give the corresponding degenerate degrees. We parameterize all the points on a quadric cone by a twisted cubic lying on the cone, and obtain a parametric coefficient matrix of the equations for estimating the fundamental matrix. By analyzing the coefficient matrix, we get the rank of it, i.e. the degenerate degree of the given configuration. It gives a more intuitive degeneracy and the degenerate degrees of the configurations, and reveals the full picture of the degeneracies on a cone.

1. Introduction

Fundamental matrix is the algebraic representation of epipolar geometry between two views. It entirely contains the geometric information that is necessary for establishing projective relation of two views. Furthermore, it plays a crucial role in many applications such as image matching, camera self-calibration, and 3-D reconstruction, etc. [1–4] Therefore, it is important to estimate the fundamental matrix accurately and robustly. However, inherent ambiguities occasionally occur if all space points and cameras lie on a “degenerate configuration” or “critical configuration” [5–7].

In two views, a set of cameras and space points is said to be degenerate for the determination of the fundamental matrix if the projective image points fail to uniquely determine the epipolar geometry [1,5,6,8,9]. Generally, the coefficient matrix of the equations for computing fundamental matrix have a rank no greater than 7 in degenerate configurations [1,10]. Moreover, it has been proved that a degenerate configuration is also unstable [6,11]. Configurations close to the degenerate ones are unstable in the sense that small amounts of noise in the data may change the estimation drastically, which would make the estimation of fundamental matrix unusable. Therefore, it is important to keep aware of the degenerate configurations in estimation algorithms. In addition, in order to obtain robust estimation of fundamental matrix [10], we need to know to what extent the degeneracies are degenerate.

However, much emphasis has been placed on the conditions of degeneracies and the detection of degeneracies in the previous works, but fewer on the degenerate degrees. Nevertheless, this is an important problem that needs an in-depth study for the sake of robust estimation and the effectiveness of estimation algorithms. In practice, the degenerate degrees differ in different configurations, e.g. the degenerate degrees in the planar scene [10] and a twisted cubic [12]. The reason for the difference in degenerate degrees is that the ranks of the coefficient matrices of the equations for computing fundamental matrix are different. And the rank could be used to indicate the degenerate degree of a critical configuration. In [12], Wu et al. analyzed all the possible degeneracies caused by a twisted cubic and gave

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corresponding degenerate degrees. In [5], a complete classification of degenerate configurations is given including numbers of conjugate solutions, however, there are some conflicts in the conclusions. For instance, it said that, the number of conjugate configurations is 1 when the camera centers lie on different generators of a cone and all the space points belong to this cone, while the points could all lie on a twisted cubic on the cone in this case, which means that the rank of coefficient matrix could be 5 [12], i.e. the number of conjugate solutions could be infinite. This critical discussion of the presented studies indicates that this problem of degeneracy degrees is still far from being solved satisfactorily, and suggests a more detailed study. However, for a non-trivial degenerate configuration, there is no direct analysis of the degenerate degree in the literature.

In this paper, we discuss the degeneracies from a quadric cone for projective reconstruction in two views and give the corresponding degrees. Unlike the previous works most of which analyzed the degeneracies based on the properties of fundamental matrix, we propose a direct way of analyzing the coefficient matrix of the equations for estimating the fundamental matrix to obtain the details of degenerate configurations on a quadric cone. Firstly, we parameterize all the points on a quadric cone by a twisted cubic belonging to it, and then we obtain a parametric coefficient matrix for estimating fundamental matrix. Using algebraic method, we could analyze the ranks of the coefficient matrices in different configurations, i.e. the degenerate degrees. It gives a more intuitive degeneracy and the degenerate degrees of the degenerate cases caused by a quadric cone. The remainder of this paper is organized as follows. To start with, we describe some definitions in Section 2 and show the relationship between a quadric cone and a twisted cubic belonging to it. Then, in Section 3, we analyze the degeneracy caused by a general configuration on a cone and discuss some special degenerate configurations. Finally, conclusions are given in Section 4.

2. Preliminaries

2.1. The camera model

The camera model used in this paper is a perspective camera that is represented in a homogeneous coordinate system. The relationship between a space point M and its projection m on an image plane is given by $m \approx PM$, where P is a rank-3 matrix of size 3×4 called the camera matrix and the symbol "$\approx$" means equality up to a scale. Let O denote the camera optical center, we have equation $PO \equiv 0$.

2.2. The fundamental matrix

Let $m_i, m_i' (i = 1 \ldots n)$ be the matching image points of the same space point M_i in two images. Then they satisfy the epipolar constraint [1]:

$$m_i'^T F m_i = 0; \quad i = 1 \ldots n \tag{1}$$

where F is a 3×3 matrix of rank 2, namely the fundamental matrix. Let $m_i \approx (u_i, v_i, w_i)^T$ and $m_i' \approx (u_i', v_i', w_i')^T$, if we denote the elements of F in row-major order by a 9-vector f, eq. (1) could be expressed as

$$Gf = \begin{bmatrix} u_1'u_1 & u_1'v_1 & u_1'w_1 & v_1'u_1 & v_1'v_1 & v_1'w_1 & w_1'u_1 & w_1'v_1 & w_1'w_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_n'u_n & u_n'v_n & u_n'w_n & v_n'u_n & v_n'v_n & v_n'w_n & w_n'u_n & w_n'v_n & w_n'w_n \end{bmatrix}_{n \times 9} f = 0 \tag{2}$$

G is the coefficient matrix of size $n \times 9$. Generally, if there are sufficiently many corresponding points in two images, then the rank of G is 8, and the fundamental matrix will be uniquely determined up to a scale by solving Eq. (2). However, if there are too few image corresponding points or all the points are lying on some special configurations, the rank of matrix G will be less than 8 and the solution for fundamental matrix will not be unique. These are degeneracies and the point configurations are called critical configurations. The rank of coefficient matrix G indicates the degenerate degree of a critical configuration.

2.3. The quadric cone and twisted cubic

The locus of points X in 3-dimensional projective space satisfying the equation

$$X^T C X = 0 \tag{3}$$

is a quadric, where C is a symmetric matrix of size 4×4. If the rank of C is 3, the locus is a quadric cone. All the generators of a cone pass through the vertex and there is only one generator through any other point of the cone.

The twisted cubic is the curve in 3-dimensional projective space on which the points satisfy the parametric equation:

$$(X, Y, Z, T)^T \approx A(\theta^3, \theta^2, \theta, 1)^T \tag{4}$$

where A is a non-singular 4×4 matrix. Six points with no three collinear and no four coplanar could uniquely determine a twisted cubic. The following relationship between a twisted cubic and a quadric cone shown in [13] is needed in this paper.

Lemma 1. The rays of the central projection of a twisted cubic from any of its points form a quadric cone.

By Lemma 1, we can conclude that all the generators of the cone are chords of the twisted cubic, and any point on the cone can be represented as a linear combination of the vertex and some other point on the twisted cubic. Therefore, on a given quadric cone, we can always find a twisted cubic passing through the vertex and all the generators of the cone are chords of the twisted cubic.