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Soliton perturbation theory of Biswas–Milovic equation

Russell Kohl[a,*], Ramazan Tinaztepe[b], Abhinandan Chowdhury[c]

[a] Department of Mathematics and Computer Science, University of Maryland at Eastern Shore, MD 70504, USA
[b] Zirve University, Kizilhisar Campus, Gaziantep 27260, Turkey
[c] Department of Mathematical Sciences, Delaware State University, DE 19901, USA

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ABSTRACT

This paper studies the perturbed Biswas–Milovic equation by the aid of soliton perturbation theory. The adiabatic variation of the soliton parameters is derived from the modified integrals of motion. The velocity of the soliton is also obtained when these perturbation terms are turned on. There are four types of nonlinear media that are taken into consideration.

1. Introduction

The theory of optical solitons has been one of the widely popular topics of research for the last few decades [1–28]. Various models for light propagation through different materials, e.g., isotropic non-Kerr material were studied and their advantages and disadvantages are analyzed extensively. Among them, Nonlinear Schrödinger's Equations (NLSE), which is a well-known soliton support system, attracted the attention of the leading researchers, and consequently, a number of excellent analytical results have been published. Most of these results are further investigated for better understanding of the fundamental phenomena associated with propagation of nonlinear waves obtained during the years – mainly because of its excellent performance in modeling the propagation of light pulses in fiber optics [6–15,27,28]. It also provides the opportunity to investigate the quasi-particle behavior of soliton which is very useful for understanding the fundamental phenomena associated with the propagation of nonlinear waves.

In order to model the dynamics of optical soliton propagation for trans-oceanic and trans-continental distances, an improved model is the Biswas–Milovic equation (BME) that was first introduced in 2010 [2]. This model accounts for the departure from perfection for the dynamics of soliton propagation through optical fibers. There are several detrimental effects that are inevitable. These include the errors due to the imperfections of the cylindrical geometry of the fibers, randomness of the injection of the pulses at the initial end of the fiber and others. Therefore the group velocity dispersion, evolution of the pulses will not be quite governed by the NLSE. Instead a generalized version of the NLSE, namely the BME is a model that is closer to reality.

This paper will address this model in presence of several perturbation terms that appears in fiber optics. The conservation laws will be revisited. The adiabatic variation of the soliton parameters will be discussed. These variations will be studied in presence of several specific

* Corresponding author.
E-mail address: rkohl81@gmail.com (R. Kohl).

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perturbation terms. There are four types of nonlinear fibers that will be taken into account, in this paper. These are the Kerr law, power law, parabolic law, and dual-power law. The results are in terms of special functions and thus “closed form” expressions for the parameter dynamics are displayed.

2. Mathematical analysis

BME first appeared in [1] as the following dimensionless form (for $m \geq 1$) of the generalized NLSE:

$$i(q^m)_t + a(q^m)_{xx} + bF(|q|^2)q^m = 0, \tag{2.1}$$

where $q = q(x, t)$ is a complex valued function. The coefficients a and b represent the coefficients of group velocity dispersion (GVD) and nonlinearity, respectively. Eq. (2.1) is a nonlinear, non-integrable partial differential equation where non-integrability is not necessarily associated with the nonlinear term present in the equation. F is a real-valued algebraic function and in order to satisfy the necessary condition of having smoothness of the complex function $F(|q|^2)q$, the function $F(|q|^2)q$ is considered to be k times continuously differentiable [1].

2.1. Integrals of motions

Just like the regular NLSE, BME also has at least three conserved quantities – they are the energy (E), linear momentum (M) and Hamiltonian (H), which are given as follow [1]:

$$E = \int_{-\infty}^{\infty} |q|^2 dx$$

$$M = i\int_{-\infty}^{\infty} (q^* q_x - q q_x^*) dx$$

and

$$H = \int_{-\infty}^{\infty} [a|q_x|^2 - bf(l)] dx,$$

where $f(I) = \int_0^I F(\xi) d\xi$, with the intensity $I = |q|^2$.

2.2. Perturbation theory

Due to the presence of perturbation terms, the energy and the momentum do not remain conserved. In fact, they undergo adiabatic changes that lead to the adiabatic deformation of the soliton amplitude, width and a slow change in the velocity.

In this section, following perturbed BME is going to be studied:

$$i(q^m)_t + a(q^m)_{xx} + bF(|q|^2)q^m = i \in R[q, q^*]. \tag{2.2}$$

In Eq. (2.2), R is a spatio-differential or integro-differential operator. The perturbation parameter ϵ (with $0 < \epsilon \ll 1$) is known as the relative width of the spectrum that arises due to quasi-monochromaticity [5,28].

When the perturbation term is introduced, the integrals of motion are modified and these modified integrals of motion can be used to derive the adiabatic variation of soliton energy (E)

$$\frac{dE}{dt} = \frac{\epsilon}{m}\int_{-\infty}^{\infty} \frac{q^* R + q R^*}{|q|^{2m-2}} dx. \tag{2.3}$$

The variation of the soliton frequency (κ) can be obtained from the adiabatic variation of the linear momentum. The expression is as follows:

$$\frac{d\kappa}{dt} = -\frac{\epsilon\kappa}{mE}\int_{-\infty}^{\infty} \frac{q^* R + q R^*}{|q|^{2m-2}} dx + \frac{i\epsilon}{2mE}\int_{-\infty}^{\infty} \left[\left\{q^* \frac{\partial}{\partial x}\left(\frac{R}{q^{m-1}}\right) - q\frac{\partial}{\partial x}\left(\frac{R^*}{q^{*m-1}}\right)\right\} - \frac{q_x^* q^{*m-1} R - q_x q^{m-1} R^*}{|q^{2m-2}|}\right] dx. \tag{2.4}$$

And, the change in the velocity of the soliton due to the presence of perturbation term

$$v = -2am\kappa + \frac{\epsilon}{mE}\int_{-\infty}^{\infty} \frac{x(q^* R + q R^*)}{|q|^{2m-2}} dx. \tag{2.5}$$

In this paper, the following perturbation terms are considered – they are all extensively studied in the context of fiber optics and optical solutions:

$$\begin{aligned} R = {} & \delta|q|^{2m} q + \alpha q_x + \beta q_{xx} - \gamma q_{xxx} + \lambda(|q|^2 q)_x + \theta(|q|^2)_x q + \rho|q_x|^2 q - i\xi(q^2 q_x^*)_x - i\eta q_x^2 q^* - i\zeta q^*(q^2)_{xx} - i\mu(|q|^2)_x q - i\chi q_{xxxx} - i\psi q_{xxxxxx} \\ & + (\sigma_1 q + \sigma_2 q_x)\int_{\infty}^{x} |q|^2 ds. \end{aligned} \tag{2.6}$$

For the sake of completeness, brief introduction of the coefficients in Eq. (2.6) is provided below. For complete details, readers may refer to [5,7,8,27,28]. δ is the coefficient of nonlinear damping or amplification depending on its sign. Also, β is the bandpass filtering term.