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Fleet algorithm for X-ray pulsar profile construction and TOA solution based on compressed sensing

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ABSTRACT

X-ray pulse profile and time of arrival (TOA) are the two important physical quantities for pulsar navigation. With the standard and integrated X-ray pulse profiles modeled, X-ray pulse profile construction is studied and TOA is solved using compressed sensing (CS) technology. The observation matrix and waveform complete dictionary are mainly examined. A column vector-based matching pursuit algorithm is presented. The feasibility of obtaining X-ray pulse profile construction by compressed sensing technology is verified by numerical simulation. Compared with the X-ray pulse profile construction method based on epoch folding, the proposed method exhibits improved real-time performance, and its detection time for integrated X-ray pulse profile could be reduced by one order of magnitude. This proposed method can also solve for TOA solution and construct the X-ray pulse profile simultaneously, which is essential to improve pulsar navigation efficiency.

1. Introduction

Pulsar is considered a "lighthouse" owing to its stable frequency and unique location in the universe, which can help determine the position, velocity, attitude, and time for spacecraft navigation [1]. The key to X-ray pulsar navigation is the time of arrival (TOA), which can be determined by X-ray pulse profile construction. The traditional method uses epoch folding to obtain the integrated X-ray pulse profile [2,3], which is compared with the standard X-ray pulse profile for solving TOA [4–6]. The disadvantages of this technique include longer observation time and lower signal-to-noise ratio (SNR). Obviously, this method cannot meet the requirements of real time and high precision for spacecraft navigation. A signal reconstruction algorithm based on compressed sensing provides a new idea for generating an X-ray pulse profile [7–10]. This concept is first applied in X-ray pulse profile reconstruction based on compressed sensing in [7], where, however, the observation matrix does not respond to the actual process of X-ray photon detection. The author merely reconstructed the X-ray pulse profile but disregarded the solution of TOA.

In view of the above analysis, this study is organized as follows:

(1) In Section 2, the process of epoch folding is analyzed. The mathematical models of the standard X-ray pulse profile and the integrated X-ray pulse profile are established.
(2) Section 3 provides the optimization of the X-ray pulse profile model based on compressed sensing. The X-ray pulse profile waveform redundant dictionary and the observation matrix are included. The matching pursuit algorithm based on the atomic column vector is also presented.
(3) The standard X-ray pulse profile, integrated X-ray pulse profiles based on epoch folding, and X-ray pulse profile based on compressed sensing are simulated numerically in Section 4.
(4) The main conclusions are stated as follows in the last section. The proposed technique using compressed sensing generates an X-ray pulse profile more quickly compared with the traditional method. The mathematical model provides a new approach to solving the issues of real time and TOA accuracy for pulsar navigation.

2. X-ray pulse profile

An X-ray pulse profile is a curve of X-ray photon flux density detected from a pulsar with time as the variable. This profile reflects the characteristics of the pulsar. Different pulsar X-ray pulse profiles have different amplitudes and periods, among others.

2.1. Standard X-ray pulse profile mathematical model

The total number of X-ray photons detected is given by

$$I(t) = \int_0^t \lambda(\tau) d\tau + F(0) \tag{1}$$

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where $F(0) = 0$ is the number of X-ray pulsar X-ray photons at time 0. $\lambda(t)$ represents the pulsar X-ray photon flux density. Within the time $[t, s]$, the detected number of X-ray photons is described as:

$$I_t^s = \int_t^s \lambda(\tau)d\tau \tag{2}$$

$\lambda(t)$ consists of two parts, as shown below:

$$\lambda(t) = \lambda_b + \lambda_p(t) \tag{3}$$

where λ_b is a constant representing the X-ray photon background noise density. $\lambda_p(t)$, which represents the X-ray photon density, can be described as follows:

$$\lambda_p(t) = \bar{\lambda}_p \cdot u(\phi(t)) \tag{4}$$

where $\bar{\lambda}_p$ denotes the average density of pulsars. The periodic function $u(\phi(t))$ is defined as the profile function. $\phi(t)$ is the time phase, $\phi \in [0, 1)$, and $\int_0^1 u(\phi)d\phi = 1$.

$$\phi(t) = \phi_0 + \int_{t_0}^{t} fre(\tau)d\tau \tag{5}$$

where ϕ_0 is the initial phase, and $fre(t)$ represents the pulse frequency. When the rate of change of frequency and Doppler shift are squared, $fre(t)$ can be expressed as:

$$fre(t) = fre_p(t) \cdot \left(1 + \frac{\nu(t)}{c}\right) \tag{6}$$

where $fre_p(t)$ is the pulsar rotation frequency, and $\nu(t)$ is defined as the constant ν. The phase function $\phi(t)$ is given by

$$\phi(t) = \phi_0 + \left[\left(1 + \frac{\nu(t)}{c}\right) \cdot (t - t_0) \cdot fre_p(t)\right] \tag{7}$$

Thus, the standard mathematical model of the X-ray pulse profile is expressed as:

$$\lambda(t) = \lambda_b + \bar{\lambda}_p \cdot u\left(\phi_0 + \left(1 + \frac{\nu(t)}{c}\right) \cdot (t - t_0) \cdot fre_p(t)\right) \tag{8}$$

When the parameters λ_b, $\bar{\lambda}_p$, ϕ_0, $u(\phi)$ are determined, the X-ray pulse profile can be constructed.

2.2. Mathematical model of the integrated X-ray pulse profile

Within the time interval δt, the probability of obtaining the number of X-ray photons, $f(i, j)$, is written as:

$$P(f(i,j), \delta t) = \frac{\left(\left(\int_{(j-1)\cdot\delta t}^{j\cdot\delta t} \lambda(t)dt\right)^{f(i,j)} . \exp\left(-\int_{(j-1)\cdot\delta t}^{j\cdot\delta t} \lambda(t)dt\right)\right)}{f(i,j)!} \tag{9}$$

Within the time interval δt, the expectation and the variance of the number of X-ray photons, $f(i, j)$ are presented as follows:

$$E[f(i,j)] = var[f(i,j)] = \int_{(j-1)\cdot\delta t}^{j\cdot\delta t} \lambda(t)dt \tag{10}$$

As $\delta t \to 0$, Eq. (10) can be approximated by

$$E[f(i,j)] = var[f(i,j)] = \lambda(t_{i,j})\delta t \tag{11}$$

The pulsar X-ray density is always in the order of 10^{-5} phs$(\text{cm}^2\,\text{s})^{-1}$. To obtain a higher SNR X-ray pulse profile, the pulsar has to be observed for an extended period and pulsar X-ray photons function has to be generated by epoch folding [3,4].

Assuming t_j is the intermediate time in the jth bin, after superimposition and normalization, the detected value of the integrated X-ray pulse profile is:

$$y(t_j) = \frac{1}{N \cdot \delta t} \sum_{i}^{N} f(i,j) \tag{12}$$

During measurement of pulsar X-ray photons, the X-ray pulse profile introduces the random noise given by

$$\bar{y}(t_j) = y(t_j) + \nu(t_j) \tag{13}$$

where, $\nu(t_j)$ is the random noise with the distribution $\nu(t_j) \sim (0, \sigma^2)$ at time t_j.

Theoretically, when the observation time $t_f \to \infty$, the standard X-ray pulse profile $x(t)$ can be obtained. Thus, a longer time is required to obtain the X-ray pulse profile using epoch folding. The current objective is to obtain an X-ray pulse profile within a short period. After epoch folding, we determine the observations $y(t)$ for the X-ray pulse profile $x(t)$. The integrated X-ray pulse profile $x(t)$ is then reconstructed based on compressed sensing from the measured values of $y(t)$.

3. X-ray pulse profile construction based on compressed sensing

3.1. Compressed sensing

Compressed Sensing is a very efficient and fast growing signal recovery frame-work. The basic principle of CS theory is that when the image of interest is very sparse or highly compressible in some basis, relatively few well-chosen observations suffice to reconstruct the most significant nonzero components. The CS model is described as:

$$\hat{x} = \min ||x||_0 \quad \text{s.t.} \quad y = \Phi \cdot f = \Phi \cdot \Psi \cdot x = \Theta \cdot x \tag{14}$$

where $Y \in Z^M$ is an observation vector. $f \in R^N$ is an unknown signal, which can be sparsely represented as $f = \Psi \cdot x$ in an orthonormal basis Ψ. If there are only $K(K \ll N)$ non-zero components of x, f is defined as being K-sparse. Φ denotes a $M \times N(M \ll N)$ matrix called measurement matrix. Θ is a sensing matrix compounded by Φ and Ψ.

In order to successfully reconstruct a signal with incomplete measurements, Θ must satisfy a special property called the restricted isometry property (RIP); that is, for all K-sparse $x \in R^N$, a constant $\delta_k \in (0, 1)$ exists so that

$$1 - \delta_k \le \frac{||\Theta x||_2^2}{||x||_2^2} \le 1 + \delta_k \tag{15}$$

There are many optimal methods to solve the problem described in Eq. (14), such as basis pursuit (BP), matching pursuit (MP), iterative thresholding, and total variation (TV) specially for signal reconstruction.

From the above model, we can see that CS contains three critical parts: signal sparsity, random observation and recovery algorithm.

3.2. Pulse signal observation

Assuming that the standard X-ray pulse profile is $p(n)$ and the integrated X-ray pulse profile is $x(n)$, the value of the observed X-ray pulse profile is $y(m)$, for $n = 1, 2, \ldots, N$ and $m = 1, 2, \ldots, M$, $M \le N$.

Pulsar detection can be considered as random sampling from discrete integrated X-ray pulse profile $x(n)$. After detection in several periods, the observation vector $y(m)$ can be obtained by epoch folding.

$$y(m) = \Phi \cdot x(n) \tag{16}$$