Optik 124 (2013) 6661–6665

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Coupled tapering/uptapering of soliton pairs in nonlinear media

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ARTICLE INFO

Article history:
Received 3 January 2013
Accepted 20 May 2013

Keywords:
All-optical manipulation
Self-focusing
Spatial solitons tapering/uptapering

ABSTRACT

Phenomenon of coupled tapering/uptapring of two mutually incoherent beams coaxially co-propagating in a nonlinear medium with small gain or loss has been investigated in this paper using standard parabolic equation approach (PEA) and the results are compared with the results obtained by Beam Propagation Method (BPM), i.e., by direct simulations of the underlying Nonlinear Schrödinger Equation (NLSE). The PEA results are shown to be in excellent agreement with the BPM results. It is seen that both beams of the pair induce uptapering in each other in presence of losses and tapering in presence of gain. When the medium offers gain to the first beam and losses to the other, both beams taper. When the medium offers gain/absorption to only one of the two beams, the beam undergoes self-tapering/self-uptapering and induces a taperd/uptaperd waveguide. The other beam (for which the medium is lossless) uptapers/tapers due to the taperd/uptaperd waveguide created by the first beam.

1. Introduction

When two optical beams propagate in a nonlinear medium, they manipulate each other as their propagation is coupled through nonlinearity. Coupled propagation in nonlinear media has been a subject of high interest as it is indispensable in all-optical switching devices (see for example, Refs. [1–5]).

In addition to coupled propagation, all-optical manipulations of light are of high importance owing to their potential applications in the fields ranging from communication to computing. Hence, self-tapering/uptapering of optical beams should be of great importance as it is the only means of all-optical control of width of an optical beam without using any fabricated structure [6]. Moreover, a tapering/uptapering beam can induce a tapered/uptapered wave guide for a low power signal beam.

It is known that when a solitonic beam enters into a nonlinear medium which has small absorption or gain, it tapers or uptapers depending upon the initial conditions [7]. Self-tapering/uptapering of solitons has been predicted/investigated in past for one solitonic beam in different media like, Kerr medium [6] saturable Kerr medium [7,8] in elliptic core fiber [9] and cubic-quintic medium [10]. Recently, two beam tapering/uptapering has also been investigated in a Kerr medium [11].

Though, important from the point of view of basic as well as applied research, the phenomenon of self-tapering/uptapering could not catch the attention which it deserves. Only a couple of groups (Snyder et al., Sodha et al., Medhekar et al.) have worked on this phenomenon. This might be due to the use of approximate methods in all previous works. Confirmation of the results of approximate method by direct simulation of the underlying NLSE would make them reliable among scientific community.

In view of it, for the first time, we have investigated coupled tapering/uptapering phenomenon by direct simulations of the underlying Nonlinear Schrödinger Equation (NLSE).

Approximate method of Ref. [11] has also been used to investigate the same phenomenon which is based on well known parabolic equation approach (PEA). It is shown that the predictions of the PEA are in excellent agreement with those obtain by direct simulations. The main aim of the present paper is to establish reliability of the results reported on tapering/uptapering phenomenon using approximate methods. The paper is being published in the hope that it would stimulate further theoretical and experimental research on self-tapering/uptapering which is the only means of all-optical manipulation of beam widths without using any fabricated structure and which is a partially explored phenomenon.

As done in Ref. [11], we obtain propagation equations of the two coupled beams using approximate method (PEA). We then investigate coupled tapering/uptapering of soliton pairs in absorbing/gain medium using both PEA and BPM for different physical situations.

2. Propagation equations using PEA

The intensity distribution of two coaxial co-propagating 1-D Gaussian beams of angular frequencies ω_1 and ω_2

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0030-4026/$ – see front matter
http://dx.doi.org/10.1016/j.ijleo.2013.05.108

respectively along the z-axis may be expressed as; $A_1^2(Z) = (E_{01}^2/f_1)\exp(-S^2/(r_1^2 f_1^2))$ and $A_2^2(Z) = (E_{02}^2/f_2)\exp(-S^2/(r_2^2 f_2^2))$ where A_1 and A_2 are the space dependent real amplitudes of the electric vectors of two beams of angular frequencies ω_1 and ω_2 and s is the transverse variable coordinate. r_1, r_2 represent dimensions of these beams at $Z=0$. f_1, f_2 are the dimensionless beam width parameters of the beams which are initially equal to 1 (at $Z=0$).

The beam width parameters f_1 and f_2 change as per beams evolution with the propagation distance and hence, $r_1 f_1$ and $r_2 f_2$ give dimensions of the beams at a distance Z. When the beams propagate through the nonlinear medium, they modify the dielectric constant of the medium as $\varepsilon(\omega_1) = \varepsilon_{10} + \phi_1(A_1 A_2)$ and $\varepsilon(\omega_2) = \varepsilon_{20} + \phi_2(A_1 A_2)$ where ε_{10} and ε_{20} are the linear part of the dielectric constants at frequencies ω_1 and ω_2 respectively and ϕ_1 and ϕ_2 are the nonlinear parts. For Kerr type nonlinear medium, ϕ_1 and ϕ_2 may be expressed as $\phi_1 = (\alpha_1 A_1^2 + k\alpha_2 A_2^2)$, $\phi_2 = (k\alpha_1 A_1^2 + \alpha_2 A_2^2)$ [11,12] here α_1 and α_2 are constants with their ratio equal to the ratio of the nonlinear coefficients of the medium at frequencies ω_1 and ω_2 respectively ($\alpha_j A_j^2$; $j=1, 2$ is dimensionless electric field intensity), k is coupling or interaction coefficient of the two beams that depends on the experimental conditions.

Following the mathematical treatment of Ref. [11], i.e., using PEA, coupled propagation equations for the two beams are obtained as;

$$\frac{\partial^2 f_1}{\partial Z^2} = \frac{1}{k_1^2 r_1^4 f_1^3} - \frac{\alpha_1 E_{02}^2}{\varepsilon_{10} r_1^2 f_1^2} - \frac{\kappa \alpha_2 E_{02}^2 f_1}{\varepsilon_{10} r_2^2 f_2^3} \quad (1)$$

$$\frac{\partial^2 f_2}{\partial Z^2} = \frac{1}{k_2^2 r_2^4 f_2^3} - \frac{\alpha_2 E_{02}^2}{\varepsilon_{20} r_2^2 f_2^2} - \frac{\kappa \alpha_1 E_{01}^2 f_2}{\varepsilon_{2_0} r_1^2 f_1^3} \quad (2)$$

Here $\alpha_j = n_{oj} n_2$; $n_{oj} = \sqrt{\varepsilon_{jo}}$ is the linear refractive index of the medium and n_2 is the nonlinear coefficient, k_1 and k_2 are the propagation constants. Thus Eqs. (1) and (2) can be written as;

$$\frac{\partial^2 f_1}{\partial Z^2} = \frac{1}{k_1^2 r_1^4 f_1^3} - \frac{n_2 E_{01}^2}{n_{0_1} r_1^2 f_1^2} - \frac{\kappa n_{02} n_2 E_{02}^2 f_1}{n_{01}^2 r_2^2 f_2^3} \quad (3)$$

$$\frac{\partial^2 f_2}{\partial Z^2} = \frac{1}{k_2^2 r_2^4 f_2^3} - \frac{n_2 E_{02}^2}{n_{02} r_2^2 f_2^2} - \frac{\kappa n_{01} n_2 E_{01}^2 f_2}{n_{02}^2 r_1^2 f_1^3} \quad (4)$$

The set of coupled Eqs. (3) and (4) governs the evolution of beams' widths of the two beams with the propagation distance in a loss less medium.

If the medium is not loss less for the two beams, i.e., if the medium is having finite losses or gains, the intensity distribution of two coaxial co-propagating beams may be rewritten as;

$A_1^2(Z) = ((E_{01}^2\gamma_1)/f_1)\exp(-S^2/(r_1^2 f_1^2))$ and $A_2^2(Z) = ((E_{02}^2\gamma_2)/f_2)\exp(-S^2/(r_2^2 f_2^2))$ where, $\gamma_j = \exp(K_j Z)$ is the loss/gain parameter, a positive K_j signifies gain, while a negative K_j signifies loss. Starting with above space dependent real amplitudes of the electric vectors and following the same mathematical procedure, Eqs. (3) and (4) are modified as;

$$\frac{\partial^2 f_1}{\partial Z^2} = \frac{1}{k_1^2 r_1^4 f_1^3} - \frac{n_2 E_{01}^2 \gamma_1}{n_{01} r_1^2 f_1^2} - \frac{\kappa n_{02} n_2 E_{02}^2 \gamma_2 f_1}{n_{01}^2 r_2^2 f_2^3} \quad (5)$$

$$\frac{\partial^2 f_2}{\partial Z^2} = \frac{1}{k_2^2 r_2^4 f_2^3} - \frac{n_2 E_{02}^2 \gamma_2}{n_{02} r_2^2 f_2^2} - \frac{\kappa n_{01} n_2 E_{01}^2 \gamma_1 f_2}{n_{02}^2 r_1^2 f_1^3} \quad (6)$$

3. Beam Propagation Method (BPM)

In BPM, the underlying propagating equations for two coupled beams co-propagating in a loss less/gain less Kerr medium are expressed as [15]

$$2ik_1 \frac{\partial E_1}{\partial Z} + \frac{\partial^2 E_1}{\partial s^2} + 2\left(\frac{k_1^2 n_2}{n_{01}}\right) \times (|E_1|^2 + k|E_2|^2)E_1 = 0 \quad (7)$$

$$2ik_2 \frac{\partial E_2}{\partial Z} + \frac{\partial^2 E_2}{\partial s^2} + 2\left(\frac{k_2^2 n_2}{n_{02}}\right) \times (|E_2|^2 + k|E_1|^2)E_2 = 0 \quad (8)$$

The solitonic solutions of Eqs. (7) and (8) in BPM are given by;

$$E_1 = E_{01}\,\mathrm{sech}\left(\frac{S}{r_1}\right)$$

$$E_2 = E_{02}\,\mathrm{sech}\left(\frac{S}{r_2}\right)$$

here, E_{01}, E_{02} are the axial electric fields of the two beams. To know the evolution of the fields' envelopes of the two beams along the propagation direction, Eqs. (7) and (8) are solved using split step Fourier method [13,14].

However, the set of equations describing two coupled beams co-propagating in a medium with finite loss or gain is

$$2ik_1 \frac{\partial E_1}{\partial Z} + \frac{\partial^2 E_1}{\partial s^2} + \beta_1 E_1 + 2\left(\frac{k_1^2 n_2}{n_{01}}\right) \times (|E_1|^2 + k|E_2|^2)E_1 = 0 \quad (9)$$

$$2ik_2 \frac{\partial E_2}{\partial Z} + \frac{\partial^2 E_2}{\partial s^2} + \beta_2 E_2 + 2\left(\frac{k_2^2 n_2}{n_{02}}\right) \times (|E_2|^2 + k|E_1|^2)E_2 = 0 \quad (10)$$

here, β_1, β_2 signify gain or loss of the two beams depending on their positive or negative signs respectively.

4. Results and discussion

We first investigate the phenomenon using PEA. The considered parameters for solving Eqs. (3)–(6) are $f_1 = f_2 = 1$, $\omega_1 = \omega_2 = 2.7148 \times 10^{15}$ rad/s, $n_{0j} = 1.63$, and the nonlinear coefficient $n_2 = 514 \times 10^{-20}\ \mathrm{m^2/W}$ [16]. When the two beams are coming from two different laser sources, they have random phase fluctuations with respect to each other and interaction coefficient for such a case (our case) is equal to unity, i.e., $k=1$ [1]. We stress here that the described phenomenon is not limited to the above parameters and one may chose any other appropriate set of parameters.

Creation of soliton pairs is essential to investigate coupled self-tapering/uptapering phenomenon. When the widths of the two beams are given, their axial field requirement for solitonic pairing could be known by putting LHS of Eqs. (3) and (4) equal to zero [11]. For beams' widths $r_1 = r_2 = 5\ \mu$m, the axial electric fields for solitonic pairing are obtained as $E_{01} = E_{02} = \sqrt{2.912 \times 10^{13}}$ V/m. The chosen electric fields and widths correspond to the Gaussian profile shown by solid curve in Fig. 1. With these parameters, Eqs. (3) and (4) give evolution of beam widths ($W_j = r_j f_j$) with the propagation as shown in Fig. 2. In the figure, overlapping W_1 and W_2 are constant with the propagation distance which confirm the formation of a solitonic pair.

We go further and investigate coupled self-tapering and uptapering of soliton pairs. The meaning of self-tapering/uptapering has been explained in earlier literature, however, we discuss it in brief here for the sake of clarity.

Self-tapering/uptapering of optical beams is the only means of all-optical control of beam width without using any fabricated structure. It is known that a solitonic beam does not change its width (and also intensity) while propagating through a loss