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Modulation of entanglement and quantum discord for circuit cavity QED states

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ABSTRACT

The paper investigates the dynamic evolution behaviors of entanglement and quantum discord of coupled superconducting qubits in circuit QED system. We put emphasis on the effects of cavity field quantum state on quantum entanglement and quantum correlations dynamic behaviors of coupling superconducting qubits. The results show that, (1) generally speaking, the entanglement will appear the death and new birth because of the interaction between qubits and cavity field, on the contrary, this phenomenon will not appear in quantum discord. (2) When the cavity field is in coherent state, the entanglement survival time is controlled by the average photon number. The more the average photon number is, the longer survival time of entanglement is prolonged. Thus it has the benefit of keeping quantum correlations. (3) When the cavity field is in squeezed state, the squeezed amplitude parameters have controlling effects on quantum correlations including entanglement and quantum discord. On the one hand, the increase of squeezed amplitude parameters can prolong the survival time of entanglement, on the other hand, with the increase of squeezed amplitude parameters, the robustness of quantum discord is more and more superior to concurrence and is more advantage to keep the system quantum correlations. The further study results show that the increase of the initial relative phase of coupling superconducting qubits can also keep the quantum correlations.

1. Introduction

Atom cavity QED system provides a good platform for quantum information processing experiments, but it has no advantage in integrations [1,2]. In order to meet the requirements of quantum information processing and truly realize the scale and integrated control, the solid quantum system should be considered. Circuit QED system is the realization of cavity QED principle in solid field. In circuit QED, superconducting qubits act as the artificial atoms and the one dimensional superconducting transmission linear resonator acts as the microwave cavity field [3–5]. Different to natural atoms, the properties of artificial atoms can be artificially designed and controlled. In addition, the strong coupling of superconducting circuit with the cavity also can be realized even if the interference of solid environment is very strong. The newest experimental research results show that the coherent time for superconducting qubits coupled to a microwave cavity can be prolonged to 0.1 ms [6]. The recent research reports also show that superconducting qubits coupled to a microwave cavity can realize the effective quantum feedback control [7]. Through the control, the superconducting qubits will make the coherent oscillation faster, slower or more continuous. The ability that can actively suppress the decoherence will be applied a lot in quantum error correction, quantum state stabilization, entanglement generation and adaptive measurement. Presently, the new research progresses in circuit QED open up a new prospect for quantum state preparation and quantum information processing.

Quantum systems exhibit diversified correlations which have no classical counterparts. It is pointed out recently that quantum entanglement, the most well known measure of quantum correlations which plays essential roles in quantum information processing, cannot describe all the nonclassicality in the correlations. The quantum discord, which can describe quantum correlations in separable states, is an important subject to intensive theoretical studies [8–12]. In the dynamic behavior investigations of quantum and classical correlation in Markovian and non-Markovian process, people find that in Markovian environment the quantum discord decays with time in a kind of asymptotic behavior which forms a bright comparison with the possible entanglement sudden death phenomenon appears in entanglement dynamics. Obviously, if the quantum discord is treated as a quantum resource, the asymptotic decay action is more advantageous to realize the quantum information processing.

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In this paper, we model a big Josephson junction as coupling element, and we couple two superconducting charge qubits with it [13,14]. Appling the quantization method, we investigate the influences on the quantum entanglement and quantum correlations dynamics of coupled superconducting qubits when the cavity field is in coherent state or squeezed state.

2. Model

We model a big Josephson junction as coupling element and couple two superconducting charge qubits with it. The Hamiltonian for the system is given by [15]

$$H = \sum_{k=1}^{2}\left[E_k(V_{xk}) - 3E_{Jk}\cos\left(\frac{\pi\Phi_e}{\Phi_0} - \frac{\gamma}{2}\right)\cos\varphi_k\right] - E_{J0}\cos\gamma \tag{1}$$

where $E_k(V_{xk}) = E_{Ck}(n_k - C_kV_{xk}/2e)^2$ is the electrostatic energy of charge qubits, $E_{Ck} = 2e^2/(C_k + 2C_{Jk})$ and E_{Jk} is the charging energy and Josephson energy of kth qubit, and E_{J0} is the Josephson energy of the big junction. Considering the situation of quantization microwave field, the flux of the change frequency is a quantized flux $\Phi_q a^+ + \Phi_q^* a$. After some calculations, the interactive Hamiltonian between quantized magnetic field and two superconducting charge qubits can be written as

$$H_I = g(a^+\sigma_1^-\sigma_2^- + a\sigma_1^+\sigma_2^+) \tag{2}$$

$$g = -\frac{2\pi\Phi_q L_J I_{C1} I_{C2}}{\Phi_0}\sin\left(\frac{2\pi\Phi_e}{\Phi_0}\right) \tag{3}$$

We suppose that the two superconducting qubits are in the Bell state with spins anti-correlated

$$\left|\psi_Q(0)\right\rangle = \cos\theta\,|gg\rangle + e^{i\varphi}\sin\theta\,|ee\rangle \tag{4}$$

and the cavity field is initially in the superposition state of number states as

$$\left|\psi_F(0)\right\rangle = \sum_n f(n)\,|n\rangle \tag{5}$$

$f(n)$ is the probability amplitude of photon number state distribution of quantum field. So the initial state of the system can be written as $\left|\Phi(0)\right\rangle = \left|\psi_F(0)\right\rangle \otimes \left|\psi_Q(0)\right\rangle$. In the interaction picture, we can have the system's vector at any time t

$$\left|\psi(t)\right\rangle = \sum_{n=0}^{\infty}\left[x_n(t)\,|e,e,n\rangle + y_n(t)\,\left|g,g,n+1\right\rangle\right] + f(0)\cos\theta\,\left|g,g,0\right\rangle \tag{6}$$

with

$$x_n(t) = e^{i\varphi}f(n)\sin\theta\cos\left(\sqrt{n+1}gt\right) - f(n+1)\cos\theta\sin\left(\sqrt{n+1}gt\right) \tag{7}$$

$$y_n(t) = e^{i\varphi}f(n)\sin\theta\sin\left(\sqrt{n+1}gt\right) + f(n+1)\cos\theta\cos\left(\sqrt{n+1}gt\right) \tag{8}$$

3. Measure of quantum entanglement and quantum correlation

We adopt the concurrence entanglement defined by Wootters to measure the system entanglement. One can calculate that the concurrence for the initial states $\left|\Phi\right\rangle$ is [16]

$$C(t) = 2\sqrt{\rho_{14}(t)\rho_{41}(t)} \tag{9}$$

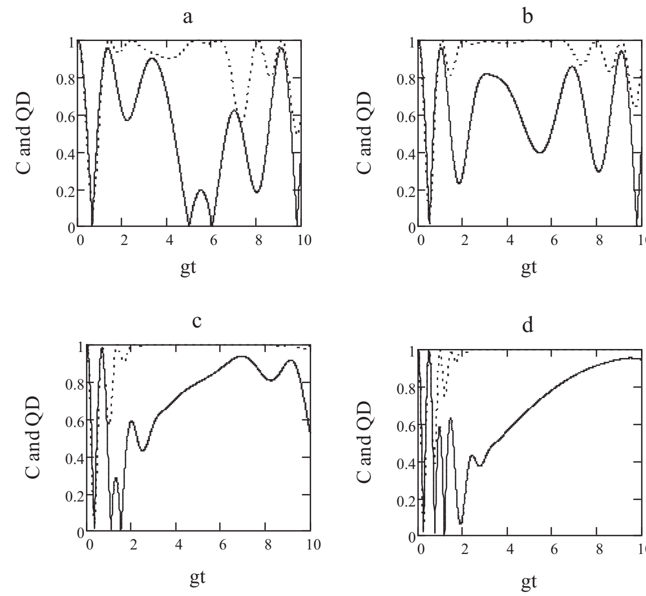

Fig. 1. The time evolution of the concurrence $C(t)$ (solid line), $QD(t)$ (dashed line) versus the scaled time gt: (a) $\bar{n} = 1$, (b) $\bar{n} = 2$, (c) $\bar{n} = 5$, (d) $\bar{n} = 10$. The two qubits are initially in the $(|gg\rangle + |ee\rangle)/\sqrt{2}$ and the field is initially in the coherent state.

A bipartite state may include not only classical correlation but also quantum correlation. We describe the quantum correlation with the quantum discord put forward by Ollivier [17,18]

$$QD(\rho_{AB}) = I(\rho_{AB}) - CD(\rho_{AB}) \tag{10}$$

where $I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$ is the quantum mutual information and $CD(\rho_{AB})$ is the classical correlation between the two subsystems. As discussed in Refs 19 and 20, the classical correlation $CD(\rho_{AB}) = \max_{\{B_k\}}\left[S(\rho_A) - S\left(\rho_{AB}\left|\rho_B\right.\right)\right]$, where $\{B_k\}$ is a set of von Neumann measurements performed on subsystem B locally, $S\left(\rho_{AB}\left|\{B_k\}\right.\right) = \sum_k p_k S(\rho_k)$ is the quantum conditional entropy, $\rho_k = (I_A \otimes B_k)\rho(I_A \otimes B_k)/Tr(I_A \otimes B_k)\rho(I_A \otimes B_k)$ is the conditional density operator corresponding to the outcome labeled by k, and $p_k = Tr(I_A \otimes B_k)\rho(I_A \otimes B_k)$ with I_A is the identity operator performed on subsystem A.

4. Discussion

First, we discuss the quantum correlation dynamics of the qubits when the field is in the coherence state. The coefficient $f(n)$ in Eq. (5) can be expressed as

$$f(n) = \exp\left(-\frac{\bar{n}}{2}\right)\frac{(\bar{n})^{\frac{n}{2}}}{\sqrt{n!}} \tag{11}$$

$\bar{n}$ is the average number of photons of the field.

The concurrence time evolution reflects the time evolution of entanglement between two qubits. We can see from Figs. 1–3 that the entanglement between two charge qubits evolves with time in oscillation behaviors, and the time evolution of concurrence appears stronger oscillation behavior with the increase of the average photons number. At some moments, the concurrence of two charge qubits is zero. That is to say at this moment, the entanglement does not exist in two charge qubits so that the entanglement death appears. Under the drive of interactions in qubits and between qubits and transmission linear, the entanglement will appear recovery phenomena.

Figs. 1–3 also show that based on the study model of the paper, the time evolution of $QD(t)$ has a big difference to concurrences. On