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Angle step selection for geometrical-optics approximation in light scattering

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Abstract

On the basis of the Shannon sampling theorem, we present a relationship between the maximal scattering angle step and the radius of particles in the calculation of light scattering intensity distribution. For the first rainbow intensity application, the relationship between the maximal scattering angle step and radius of particles is derived from that between the ripple frequency and radius and refractive index of particles using this method. For the geometrical-optics approximation, the incident angle is used to calculate the scattering intensity distribution. To get the highest speed, the maximal incident angle step is necessary. The relationship between the maximal step of incident angle and radius of particles is deduced from the maximal scattering angle step equation. As indicated by our result, the maximal step of the incident angle is not a constant and it varies with incident angle.

Keywords: Light scattering; Geometrical-optics approximation; Optical measurement; Angle step

1. Introduction

The scattering of light by a sphere has been of interest to many researchers during the past several decades [1]. To rigorously describe the theoretical scattering intensities of particles of different sizes, the Mie theory was developed more than 100 years back. To clarify the physical origins of many important effects that occur in the study of characteristics of light scattering, the Debye series [2] has been developed and employed in the study of rainbows and other phenomena. The calculations of the above two methods have been proved difficult and time consuming, and the application is restricted by the calculation speed. In such a situation, the geometrical-optics approximation [3] (GOA) was developed as a faster approximate algorithm.

When GOA is employed in real-time measurement technology, the calculation time is very important and high calculation speed is required. However, the relationship between the incident angle step and the calculation time of GOA has never been researched.

As shown in the application of GOA to industry, the step of incident angle is often a constant [3]. With large radius, the constant step of incident angle causes a discrepancy in scattering intensity distribution. If the radius is small, the constant incident angle step can cause superfluous calculation time. Therefore the right step of the incident angle should be selected for a certain radius. In this paper we present a relationship between the maximal scattering angle step and radius of particles in the calculation of light scattering intensity distribution. And then the relationship between the maximal

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step of incident angle and radius of particles is deduced from the maximal scattering angle step equation.

2. Geometrical-optics approximation [1]

For a sphere with a radius R, the dimensionless parameter is set $\alpha = 2\pi R/\lambda$, where λ is the wavelength of the incident light. From a geometrical-optics viewpoint, the dimensionless scattering intensity is defined as

$$i_j = \alpha^2 k_j^2 D \quad j = 1, 2 \tag{1}$$

where subscript 1 represents perpendicular polarized components in polarization and subscript 2 represents parallel polarized components. k_j is introduced to characterize the fraction of the emerging rays in the incident intensity

$$\begin{aligned} k_j &= \varepsilon_j && \text{for } N = 1 \\ k_j &= (1-\varepsilon_j^2)(-\varepsilon_j)^{N-2} && \text{for } N = 2, 3, 4\ldots \end{aligned} \tag{2}$$

D is usually called divergence or gain, a parameter denoting the influence of the shape of a particle on the angular intensity distribution. It can be expressed as

$$D = \frac{\sin\tau\cos\tau}{\sin\theta|d\theta'/d\tau|} \tag{3}$$

As indicated in Fig. 1, the relationship of the incident angles to the scattering angles of a sphere is described as

$$\theta' = 2\tau - 2(N-1)\tau' \tag{4}$$

where τ represents the complement of the incident angle, τ' is the complement of the internal refracted angle and it is defined as $\cos\tau' = \cos\tau/m$. N is the number of light–sphere interactions. The scattering angle can be defined in the interval $(0, \pi)$

$$\theta' = 2k\pi + q\theta \tag{5}$$

where k is an integer and $q = +1$ or $q = -1$. $\varepsilon_{1,2}$ are Fresnel reflection coefficients, defined as

$$\begin{aligned} \varepsilon_1 &= \frac{\sin\tau - m\sin\tau'}{\sin\tau + m\sin\tau'} \\ \varepsilon_2 &= \frac{m\sin\tau - \sin\tau'}{m\sin\tau + \sin\tau'} \end{aligned} \tag{6}$$

where m is the refractive index of sphere.

In terms of van de Hulst's discussion of the phase change that is due to optical length σ_l and focal lines σ_f:

$$\begin{aligned} \sigma_l &= 2\alpha[\sin\tau - (N-1)m\sin(\tau')] \\ \sigma_f &= \frac{\pi}{2}\left(N - 1 - 2k + \frac{1}{2}s - \frac{1}{2}q\right) \end{aligned} \tag{7}$$

The integers N, k, q are defined above, and s denotes $+1$ or -1, equal to the sign of $d\theta'/d\tau$. The combined phases $\sigma_{j,N}$ can finally be obtained

$$\sigma_{j,N} = \sigma_l + \sigma_f \tag{8}$$

Amplitude and phase of the scattered light may be expressed by means of the "amplitude function" $S_{j,N} = \sqrt{i_{j,N}}e^{i\sigma_{j,N}}$. From a geometrical-optics viewpoint, scattered light amplitude is considered to be the superposition of diffracted, refracted, and reflected fractions. The final amplitude functions can be finally obtained

$$S_j = S_{diff} + \sum_{N=1}^{\infty} S_{j,N} \quad j = 1, 2 \tag{9}$$

where $S_{diff} = \alpha^2 J_1(\alpha\sin\theta)/(\alpha\sin\theta)$ and J_1 is the first order Bessel function.

Suppose that the particle is illuminated by an unpolarized monochromatic beam with a wavelength of λ and an incident intensity of I_0. The scattering intensity $I(\theta)$ at distance f can finally be obtained by

$$I(\theta) = \frac{\lambda^2 I_0}{8\pi^2 f^2}(|S_1(\theta)|^2 + |S_2(\theta)|^2) \tag{10}$$

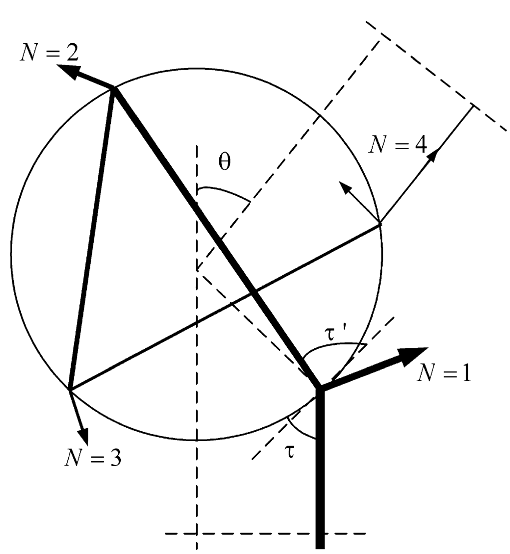

Fig. 1. Path of a light ray through the sphere according to geometrical optics.

3. Analysis of angle step

The frequency of the scattering intensity is used in the measurement of particle and refractive index. Thus, the exact frequency is purposed of the principle of the angle step selection in this paper. For the GOA method, the step of the incident angle is used to calculate the scattering intensity distribution. However, the relationship between the maximal step of incident angle and radius of particles is not obvious. Therefore the maximal step of the scattering angle is studied first.