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Comparison of two theories during self-focusing of Gaussian laser beam in thermal conduction-loss predominant plasmas

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ABSTRACT

In the present paper, self-focusing phenomenon occurring as a result of non-linear interaction of intense laser beam with thermal conduction-loss predominant plasmas is studied by following both approaches viz. paraxial theory approach and moment theory approach. Non-linear differential equations for the beam width parameters of laser beam have been set up and solved numerically in both cases to study the variation of beam width parameters with normalized distance of propagation. Effects of laser intensity as well as plasma density on the beam width parameters have also been analyzed. It is observed from the analysis that in case of moment theory approach, strong self-focusing of laser beam is observed as compared to paraxial theory approach.

1. Introduction

An efficient coupling of a high power laser beam with plasma is a topic of current research interest in many areas such as laser induced fusion and particle acceleration. In the laser plasma interaction process, various non-linear phenomena such as self-focusing, filamentation, stimulated Raman scattering, stimulated Brillouin scattering, two plasmon decay, etc. [1–3] play a crucial role. However, self-focusing continues to be a subject of great fascination due to its relevance to ionospheric radio propagation, optical harmonic generation, X-ray lasers and other important applications [4–9]. The self-focusing of laser beams, having nonuniform distribution of irradiance in a plane, normal to direction of propagation leads to nonuniform distribution of carriers along the wave-front, which further leads to a change in dielectric constant of plasma. The collisional nonlinearity occurs because of electrons acquiring temperature higher than other species on account of net effect of ohmic heating and energy lost by electrons due to collisions with heavy particles (atoms/molecules and ions) and by thermal conduction [10–15]. These analysis consider only one type of energy loss viz. collisions or thermal conduction. Most of the analysis of self-focusing are based on paraxial theory approach [10,11], which take in to account only paraxial region of the beam and thus lead to large error in the critical power. Importance of non-paraxiality in self-focusing mechanism has already been pointed out [16]. In some experiments, where solid state lasers are used, wide angle beams are generated for which the paraxial approximation is not applicable. Also, if the beam width of laser beam used is comparable to the wavelength of the laser beam, paraxial approximation is not valid. In this theory non-linear part of the dielectric constant is Taylor expanded up to second order term and higher order terms are neglected. However, the moment theory [17,18] is based on the calculation of moments and does not suffer from this defect. In moment theory approach, non-linear part of the dielectric constant is taken as a whole in calculations [19–26].

In the present paper, the self-focusing phenomenon occurring as a result of non-linear interaction of laser beam with thermal conduction-loss predominant plasmas is studied by following both approaches viz. the paraxial ray approach and the moment theory approach. Non-linear differential equations for the beam width parameters of laser beam are set up and solved numerically in both cases to study the variation of beam width parameters with normalized distance of propagation. Effects of laser intensity as well as plasma density on the behavior of beam width parameters are also analyzed.

2. Solution of wave equation

Consider the propagation of a laser beam of angular frequency ω_0 in a homogeneous collisional plasma along z-axis. The initial

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intensity distribution of beam along the wavefront at $z=0$ is given by

$$E_0.E_0^{\star}|_{z=0} = E_{00}^2 \exp\left[\frac{-r^2}{r_0^2}\right] \quad (1)$$

where $r^2=x^2+y^2$ and r_0 is initial width of the main beam and r is radial co-ordinate of the cylindrical co-ordinate system.

The slowly varying electric field E_0 of the laser beam satisfies the following wave equation.

$$\nabla^2 E_0 - \nabla(\nabla.E_0) + \frac{\omega_0^2}{c^2}\epsilon E_0 = 0 \quad (2)$$

In the WKB approximation, the second term $\nabla(\nabla . E_0)$ of Eq. (2) can be neglected, which is justified when $(c^2/\omega_0^2)|(1/\epsilon)\nabla^2 \ln \epsilon| \ll 1$, So, one can get

$$\nabla^2 E_0 + \frac{\omega_0^2}{c^2}\epsilon E_0 = 0 \quad (3)$$

and

$$\epsilon = \epsilon_0 + \epsilon_1 \quad (4)$$

where $\epsilon_0 = 1 - (\omega_p^2/\omega_0^2)$ is the linear part of the dielectric constant, $\omega_p = \sqrt{4\pi n_0 e^2/m}$ is the plasma frequency and other symbols have their usual meanings. ϵ_1 represents the non-linear part of the dielectric constant and is given by [15],

$$\epsilon_1 = \frac{\omega_p^2 \beta}{16\omega_0^2 \theta T_0^{7/2}} \frac{E_{00}^2}{f_0^2} r^2 \quad (5)$$

Electric field E_0 can be written as

$$E_0 = A(r, z)\exp[\iota\{\omega_0 t - k_0 z\}] \quad (6)$$

where $A(r, z)$ is a complex function of its argument. The behavior of the complex amplitude $A(r, z)$ is governed by the parabolic equation obtained from the wave Eq. (3) in the WKB approximation by assuming variations in the z direction being slower than those in the radial direction,

$$-2\iota k_0 \frac{\partial A}{\partial z} + \nabla_{\perp}^2 A + \frac{\omega_0^2 \epsilon_1 A}{c^2} = 0. \quad (7)$$

This equation is also known as the quasi optic equation.

3. Paraxial theory approach

Further assuming the variation of $A(r, z)$ as

$$A(r, z) = A_0(r, z)\exp[-\iota k_0 S_0(r, z)] \quad (8)$$

where $A_0(r, z)$ and S_0 are real functions of r and z (S_0 being the ekional). On substituting A in Eq. (7) and separating the real and imaginary parts of the resulting equation, the following set of equations is obtained:

$$2\left(\frac{\partial S_0}{\partial z}\right) + \left(\frac{\partial S_0}{\partial r}\right)^2 = \frac{1}{k_0^2 A_0}\nabla_{\perp}^2 A_0 + \frac{\epsilon_1}{\epsilon_0} \quad (9)$$

and

$$\frac{\partial A_0^2}{\partial z} + \left(\frac{\partial S_0}{\partial r}\right)\frac{\partial A_0^2}{\partial r} + A_0^2 \nabla_{\perp}^2 S_0 = 0. \quad (10)$$

Following [10,11,15], the solutions for Eqs. (9) and (10) can be written as

$$A_0^2 = \frac{E_{00}^2}{f_0^2}\exp\left[-\frac{r^2}{r_0^2 f_0^2}\right], \quad (11)$$

$$S_0 = \frac{r^2}{2}\beta_0(z) + \Phi_0(z) \quad (12)$$

where $\beta_0(z)=(1/f_0)(df_0/dz)$ and $k_0 = (\omega_0 \epsilon_0^{1/2})/c$. The parameter β_0^{-1} may be interpreted as the radius of the curvature of the main beam and $\Phi_0(z)$ is the phase shift, which we do not require for the further analysis as we are interested in the intensity of the laser beam rather than its phase. On substituting Eqs. (11) and (12) in Eq. (9) and on equating the coefficients of r^2 on both sides, we get the following differential equation for the beam width parameter f_0 of the laser beam

$$\frac{d^2 f_0}{d\xi^2} = \frac{1}{f_0^3} - \frac{\omega_p^2 r_0^4}{c^2}\frac{\alpha E_{00}^2}{f_0} \quad (13)$$

where $\xi = (z/k_0 r_0^2)$ is the dimensionless propagation distance and $\alpha = \beta/(16\theta T_0^{7/2})$. Eq. (13) describes the variation in the beam width parameter f_0 of a Gaussian laser beam on account of the competition between diffraction divergence and nonlinear focusing terms as the beam propagates in the thermal conduction-loss predominant plasma.

4. Moment theory approach

Eq. (7) can be written as

$$\iota\frac{dA}{dz} = \frac{1}{2k_0}\nabla_{\perp}^2 A + \chi(AA^{\star})A \quad (14)$$

where $\chi(AA^{\star})=(k_0/2\epsilon_0)(\epsilon-\epsilon_0)$ and $\epsilon=\epsilon_0+\epsilon_1$, where $\epsilon_o = 1 - (\omega_p^2/\omega_0^2)$ and ϵ_1 are the linear and nonlinear parts of the dielectric constant, respectively. Also, $k_0 = (\omega_0/c)\sqrt{\epsilon_0}$ and ω_p are propagation constant and plasma frequency, respectively. Now from the definition of the second order moment, the mean square radius of the beam is given by

$$< a^2 >= \frac{\int\int(x^2+y^2)AA^{\star}dxdy}{I_0}. \quad (15)$$

From here one can obtain the following equation.

$$\frac{d^2 < a^2 >}{dz^2} = \frac{4I_2}{I_0} - \frac{4}{I_0}\int\int Q(|A|^2)dxdy \quad (16)$$

where I_0 and I_2 are the invariants of Eq. (14) [17]

$$I_0 = \int\int |A|^2 dxdy \quad (17)$$

$$I_2 = \int\int \frac{1}{2k_0^2}(|\nabla_{\perp}|A|^2 - F)dxdy \quad (18)$$

With [18]

$$F(|A|^2) = \frac{1}{k_0}\int \chi(|A|^2)d(|A|^2) \quad (19)$$

and

$$Q(|A|^2) = \left[\frac{|A|^2\chi(|A|^2)}{k_0} - 2F(|A|^2)\right]. \quad (20)$$

For $z>0$, we assume an energy conserving Gaussian ansatz for the laser intensity [10,11,15]

$$AA^{\star} = \frac{E_{00}^2}{f_0^2}\exp\left\{-\frac{r^2}{r_0^2 f_0^2}\right\}. \quad (21)$$

From Eqs. (15), (17) and (21), it can be shown that

$$I_0 = \pi r_0^2 E_{00}^2, \quad (22)$$

$$< a^2 >= r_0^2 f_0^2 \quad (23)$$