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Rayleigh–Benard Convection in the homeotropic nematic liquid crystal by a Gaussian laser beam absorption

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ABSTRACT

The Rayleigh–Benard Convection (RBC) were studied theoretically in the homeotropic nematic liquid crystal (NLC) cell due to the absorption of a normally incident Gaussian light beam from upper side of the cell. It was shown that a convection of hydrodynamic motion in the NLC can be induced and eventually the molecular director coupling with hydrodynamics reorients the director. An external magnetic field parallel to the director decreases the director reorientation and the radial flow velocity, while the velocity in the z direction is unaffected. The results were found to be in a range that can easily be checked experimentally.

1. Introduction

Liquid crystals (LC) are interesting materials that have physical properties between conventional fluids and solids [1]. One of the most important features of the LCs which results novel properties in the LCs is the ability of the molecular director reorientation due to different mechanisms such as the optical [2], thermomechanical [3], mechanical [4], photorefractive [5], mechanisms. In all of the mentioned cases the problem is to find a suitable mechanism transforming the absorbed energy in the LC to the molecular deformation energy. The hydrodynamical motions can also be induced in the LCs [6]. There are three main mechanisms of thermal induced hydrodynamic motion: gravity or the Rayleigh–Benard Convection (RBC) [7], thermocapillary or the Marangoni effect [8] and the direct volume expansion mechanism [9]. Convective motion in the isotropic horizontal layer fluid heated from below, under the action of the gravity force is known as gravity or RBC. In the classical Rayleigh–Benard problem a sample of Newtonian fluid is subjected to an adverse thermal gradient. When the temperature difference between the upper and lower boundaries is less than a critical value, the system remains in the equilibrium and there is no flow. However, as this critical value is exceeded the onset of stationary convection is observed. This simple hydrodynamic instability occurs when the buoyancy force due to the thermal expansion near the lower surface is sufficient to overcome the opposing viscous shear force. This hydrodynamic motion is due to the variation of the fluid density and is called convective motion. This convective motion leads to the relocation of the upper cool layers with the lower hot layers and also to the circular rolls of the layers in the NLC cell [10].

Because of the anisotropy of the liquid crystal, the orientation deformation reflects back not only on the flow (viscosity anisotropy) but also on the heat transfer [11]. The coupling between the flow, temperature and director orientation allows the possibility of interesting phenomena in the NLC not seen in the conventional isotropic liquids. As a result the induced hydrodynamic motion leads to the director reorientation in the NLC [7,9,11]. For example the RBC heated from above does not occur in the homogeneous ordinary liquids, but can take place in the LCs [6].

In this report the RBC in a homeotropic NLC due to a laser Gaussian beam absorption incident normally from above is studied theoretically. When a horizontal layer of a homeotropic NLC is subjected to a thermal gradient heating from above, the thermal diffusivity anisotropy produces a heat focusing effect in the medium. The RB instability reduces drastically the critical temperature gradient [12,13]. The NLCs are sensitive to an external magnetic or electric field [14]. Depending on the magnetic field direction with respect to the director, the reorientation threshold filed will be different [12]. In fact, due to the positive anisotropy in the magnetic susceptibility this kind of fluid has the property of aligning the director in the same direction as H when the intensity of H reaches a critical value (Freedericksz transition).

This report is organized as follows. In Section 2 the velocity components in the direction r and z directions are calculated. The anisotropy of heat transfer in the induced temperature

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profile is taken into account (Section 3). The torque balance equation indicates the director reorientation (Section 4). Section 5 includes the numerical calculations of the coupled equations, and finally Section 6 concludes the results.

2. Navier–Stokes equation

The Navier–Stokes equation explains that a fluid how flows under the influence of different agents. Because of the mentioned coupling the equations of the Navier–Stokes, torque balance and temperature diffusivity should be simultaneously solved in the NLCs [15]. The following assumptions are used in deriving the mentioned equations; the Boussinesq approximation that ignores the thermal variations of the physical parameters else than the thermal dependence of the density in the buoyancy force [14]; the hard anchoring condition in the both cell walls, and the incompressibility condition $\nabla.v = 0$ where v is the flow velocity.

A horizontal layer $0 \geq Z \geq L$ of homeotropically oriented NLC (unperturbed director, $n_0 = e_z$) in cylindrical coordinate system is considered. The layer is in the gravitational field with $g = -ge_z$ and absorbs a Gaussian incident light from above. Due to the symmetry of the heat diffusivity and director orientation around the z axis $\partial/\partial\varphi$ and $v\varphi$ are zero [16]. The excited hydrodynamic motions are described by the Navier–Stokes equation as follows:

$$\rho\left(\frac{\partial v}{\partial t} + (\nabla.v)v\right) = -\nabla p + f_{\text{vis}} + f_{\text{ext}}. \quad (1)$$

The i component of f_{vis} is $f_i = \left(\frac{\partial \sigma_{ij}}{\partial x_j}\right)$ that σ_{ij} is the viscose stress tensor in the NLC where

$$\sigma_{ij} = \alpha_1 n_i n_j n_k n_m d_{km} + \alpha_2 n_j N_i + \alpha_3 n_i N_j + \alpha_4 d_{ij} + \alpha_5 n_j n_k d_{ki} + \alpha_6 n_i n_k d_{kj}, \quad (2)$$

n is the director unit vector and $\alpha_1 - \alpha_6$ are the Leslie coefficients.

$$N_i = \frac{dn_i}{dt} + \frac{1}{2}(n \times w)_i \quad (3)$$

$$d_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \quad (4)$$

where d_{ij} is the velocity – gradient tensor and N is the rate of change of the director with respect to the immobile background fluid and w is the angular velocity. These variables are written in the following form; $P = P_0 + P'$, $\rho = \rho_0(1 - \beta T)$, where, P_0, and ρ_0 are the unperturbed pressure ($P_0 = -\rho_0 gz$) and density; P' and v', are the pressure and velocity perturbations, and β is the volume expansion coefficient of the LC. The r and z components of the Navier–Stokes equation after linearization are respectively:

$$\rho_0\left(\frac{\partial v_r(r,z,t)}{\partial t}\right) = \alpha_4 \frac{\partial^2 v_r(r,z,t)}{\partial r^2} + \alpha_2 \frac{\partial^2 n_r(r,z,t)}{\partial z \partial t} + \frac{1}{2}(\alpha_3 + \alpha_4 + \alpha_6)\frac{\partial^2 v_r(r,z,t)}{\partial z^2} + \alpha_3\left(\frac{\partial^2 n_r(r,z,t)}{\partial r \partial t} + \frac{\partial n_r(r,z,t)}{\partial t}\frac{1}{r}\right) \quad (6)$$

$$\rho_0\left(\frac{\partial v_z(r,t)}{\partial t}\right) = -\rho_0 T(r,z,t)\beta g + \left(\frac{1}{r} + \frac{\partial}{\partial r}\right) \times \left(0.5(\alpha_3 + \alpha_4 + \alpha_6)\frac{\partial v_z(r,t)}{\partial r} + 0.5(-\alpha_3 + \alpha_4 + \alpha_6)\frac{\partial v_z(r,z,t)}{\partial z}\right) \quad (7)$$

3. Thermal conductivity equation

Because of the thermal anisotropy of the NLC the heat flux q_i is related by a second – order symmetric tensor to the temperature gradient $q_i = -k_{ij}\frac{\partial T_i}{\partial x_j}$ [10]. The thermal conductivity is written in the following form:

$$\rho_0 c_p\left(\frac{\partial T}{\partial t} + v.\nabla T\right) = -\nabla q + Q \quad (8)$$

where Q is the light induced heat source with the form of $\frac{P}{2\pi a^2}\exp\left[\frac{-r^2}{2a^2}\right]\exp[-\alpha_\perp z]$, p is the light power, a is the light spot size, $\alpha_\perp$ is the perpendicular cell absorption coefficient, and q is the heat flux vector.

$$q_i = -k_{ij}\frac{\partial T_j}{\partial x_j} \quad (9)$$

$$k_{ij} = k_{\text{iso}}\delta_{ij} + k_a\left(n_i n_j - \frac{\delta_{ij}}{2}\right) \quad (10)$$

$$k_{\text{iso}} = \frac{k_\parallel + k_\perp}{3} \quad (11)$$

$$k_a = k_\parallel - k_\perp \quad (12)$$

where k_{ij} is the thermal conductive tensor, k_{iso} and k_a are the isotopic and anisotopic thermal conductive and $k_\parallel$, $k_\perp$ are the parallel and perpendicular thermal conductivities, respectively. The temperature equation after linearization is as follows:

$$\rho_o c_p\left(\frac{\partial T(r,z,t)}{\partial t}\right) + v_r(r,z,t)\frac{\partial T(r,z,t)}{\partial t} + v_z\frac{\partial T(r,t)}{\partial z} = \frac{1}{r}k_\perp\left(\frac{\partial T(r,z,t)}{\partial r}\right) + k_\perp\left(\frac{\partial^2 T(r,z,t)}{\partial r^2}\right) + k_\parallel\frac{\partial^2 T(r,z,t)}{\partial z^2} + a_\perp\frac{p}{2\pi a^2}\exp\left[\frac{-r^2}{2a^2}\right]\exp[-a_\perp z] \quad (13)$$

where ρ_0 is the density, p is the pressure, v is the velocity, T is the temperature and c_p is the specific heat capacity.

4. Torque balance equation

The torque balance equation shows the director reorientation due to the all forces as hydrodynamic, Frank elasticity and external forces:

$$(n \times f)_i + e_{ijm}n_m\left[\frac{\partial F}{\partial n_j} - \frac{\partial}{\partial x_k}\frac{\delta F}{\delta(\partial n_j/\partial x_k)}\right] = 0 \quad (14)$$

$$f_i = (\gamma_1 N_i + \gamma_2 d_{ij}) \quad (15)$$

$$N_i = \frac{dn_i}{dt} + \frac{1}{2}(n \times w)_i \quad (16)$$

$$\gamma_1 = (-\alpha_2 + \alpha_3) \quad (17)$$

$$\gamma_2 = (\alpha_2 + \alpha_3) \quad (18)$$

$$F = k_1(\nabla.n)^2 + k_2(n.\nabla \times n)^2 + k_3(n \times \nabla \times n)^2 \quad (19)$$

where f is the hydrodynamical force, F is the Frank free energy and $k_1 \ldots k_3$ are the Frank elastic constants. The torque balance equation